Derivative of an Inverse

Given $f(a) = b \leftrightarrow f^{-1}(b) = a$
$f'(a) = m$

The Derivative of the inverse of f(x) at x = b is the reciprocal of the derivative of f(x) at a.

$$(f^{-1})'(b) = \frac{1}{m} = \frac{1}{f'(a)} = \frac{1}{f'(f^{-1}(b))}$$

The Derivative of an inverse at a value is the reciprocal of the derivative of the Original at the pre-image.

Proof:
$f(g(x)) = x$

$f'(g(x))\, g'(x) = 1$

$$g'(x) = (f^{-1})'(x) = \frac{1}{f'(g(x))} = \frac{1}{f'(f^{-1}(x))}$$

Integrals Resulting in Inverse Trig Functions

$$\frac{d}{dx} \arcsin u = \frac{1}{\sqrt{1 - u^2}} \frac{du}{dx}$$

$$\int \frac{du}{\sqrt{1-u^2}} = \arcsin u + C$$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \int \frac{du/a}{\sqrt{1-(u/a)^2}} = \boxed{\arcsin\left(\frac{u}{a}\right) + C}$$

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$$\frac{d}{dx} \arctan u = \frac{1}{1 + u^2} \frac{du}{dx}$$

$$\int \frac{du}{1+u^2} = \arctan u + C$$

$$\int \frac{du}{a^2 + u^2} = \int \frac{\frac{du}{a}}{a\left[1+\left(\frac{u}{a}\right)^2\right]} = \frac{1}{a}\int \frac{\frac{du}{a}}{1+\left(\frac{u}{a}\right)^2} = \boxed{\frac{1}{a}\arctan\left(\frac{u}{a}\right) + C}$$

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$$\frac{d}{dx} \operatorname{arcsec} u = \frac{1}{|u|\sqrt{u^2 - 1}} \frac{du}{dx}$$

$$\int \frac{du}{|u|\sqrt{u^2 - 1}} = \operatorname{arcsec} u + C$$

$$\int \frac{du}{|u|\sqrt{u^2 - a^2}} = \int \frac{\frac{du}{a}}{a\left|\frac{u}{a}\right|\sqrt{\left(\frac{u}{a}\right)^2 - 1}} = \frac{1}{a}\int \frac{\frac{du}{a}}{\left|\frac{u}{a}\right|\sqrt{\left(\frac{u}{a}\right)^2 - 1}} = \boxed{\frac{1}{a}\operatorname{arcsec}\left(\frac{|u|}{a}\right) + C}$$

Summary of Derivatives and Integrals involving Inverse Trig Functions:

Inverse Trigonometric Functions: Differentiation and Integration

1. $\frac{d}{dx}\left[\arcsin u\right] = \frac{1}{\sqrt{1 - u^2}} \bullet \frac{du}{dx}$	$\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$
2. $\frac{d}{dx}\left[\arccos u\right] = \frac{-1}{\sqrt{1 - u^2}} \bullet \frac{du}{dx}$	$\int \frac{-du}{\sqrt{a^2 - u^2}} = \arccos \frac{u}{a} + C$
3. $\frac{d}{dx}\left[\arctan u\right] = \frac{1}{1 + u^2} \bullet \frac{du}{dx}$	$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$
4. $\frac{d}{dx}\left[\text{arccot } u\right] = \frac{-1}{1 + u^2} \bullet \frac{du}{dx}$	$\int \frac{-du}{a^2 + u^2} = \frac{1}{a} \text{arccot} \frac{u}{a} + C$
5. $\frac{d}{dx}\left[\text{arcsec } u\right] = \frac{1}{\lvert u\rvert\sqrt{u^2 - 1}} \bullet \frac{du}{dx}$	$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \text{arcsec} \frac{\lvert u\rvert}{a} + C$
6. $\frac{d}{dx}\left[\text{arccsc } u\right] = \frac{-1}{\lvert u\rvert\sqrt{u^2 - 1}} \bullet \frac{du}{dx}$	$\int \frac{-du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \text{arccsc} \frac{\lvert u\rvert}{a} + C$

AP Calculus BC 1 Assignment 145 Wednesday, December 9, 2015 Hour Name

Non-Calculator

1. $\int \frac{5}{\sqrt{9-x^2}}\,dx =$

2. $\int \frac{3}{\sqrt{1-4x^2}}\,dx =$

3. $\int \frac{7}{16+x^2}\,dx =$

4. $\int \frac{1}{x\sqrt{4x^2-1}}\,dx =$

5. $\int \frac{t}{t^4+16}\,dt =$

6. $\int \frac{\cos x}{1+\sin^2 x}\,dx =$

7. Using the information from the table below to find $\frac{d}{dx}\left[f^{-1}(3)\right]$

x	1	2	3	4	5
f(x)	2	4	5	1	3
f '(x)	5	3	1	2	4