Notes on Bases Other than e

$f(x) = a^x$
$= (e^{\ln a})^x = e^{\ln a \bullet x}$

$f'(x) = e^{\ln a \bullet x} \ln a$
$= a^x \ln a$

$$\boxed{\frac{d}{dx}(a^x) = a^x \ln a}$$

$f(x) = \log_a x$
$= \frac{\ln x}{\ln a} = \ln x \frac{1}{\ln a}$

$f'(x) = \frac{1}{x} \bullet \frac{1}{\ln x}$

$$\boxed{\frac{d}{dx}(\log_a x) = \frac{1}{x} \cdot \frac{1}{\ln a}}$$

$\int a^x \, dx = \int e^{x \ln a} \, dx$
$= \frac{1}{\ln a} \int e^{x \ln a} \ln a \, dx = \frac{1}{\ln a} e^{x \ln a} + C = \frac{1}{\ln a} a^x + C$

$$\boxed{\int a^x \, dx = a^x \cdot \frac{1}{\ln x} + C}$$

$f(x) = x \ln x - x$

$f'(x) = x \cdot \frac{1}{x} + \ln x - 1 = 1 + \ln x - 1 = \ln x$

$$\boxed{\int \ln u \, du = u \ln u - u + C}$$

$\int \log_a u \, du = \int \frac{\ln u}{\ln a} \, du = \frac{1}{\ln a} \int \ln u \, du$

$$\boxed{\int \log_a u \, du = \frac{1}{\ln a}(u \ln u - u) + C}$$

==

$\int \sec x \, dx = \int \sec x \left(\frac{\tan x + \sec x}{\tan x + \sec x} \right) dx = \int \frac{\left(\sec x \tan x + \sec^2 x\right) dx}{\tan x + \sec x}$

This fits the Theorem: $\int \frac{du}{u} = \ln|u| + C$

$$\boxed{\int \sec x \, dx = \ln|\tan x + \sec x| + C}$$

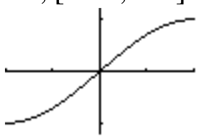
Inverse Trig Functions:

Recall that A function has an Inverse iff it is one-to-one.
If we consider the function y = sin x, it cannot have an inverse because it is not a one-to-one function over its entire domain of real numbers. But, if we restrict the domain to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, then this restricted function is one-to-one. Often this new function is written as y = Sin x. The uppercase S indicates the restricted version.
In a similar fashion, we must restrict y = cos x, y = tan x, y = cot x, y = sec x, and y = csc x to insure that they are also one-to-one before seeking their inverses.

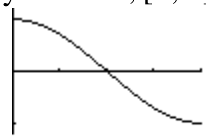
$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \leftrightarrow \arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$ $\tan\left(\frac{\pi}{4}\right) = 1 \leftrightarrow \tan^{-1}(1) = \frac{\pi}{4}$ arccos 2 → DNE

Graphs of one-to-one versions of Trigonometric Functions

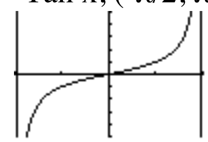
y = Sin x, [-π/2, π/2]



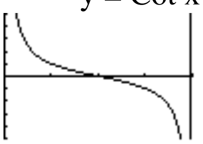
y = Cos x, [0, π]



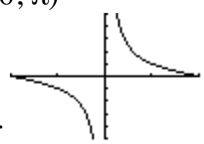
y = Tan x, (-π/2, π/2)



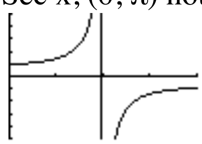
y = Cot x, (0, π)



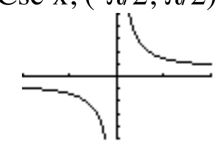
or



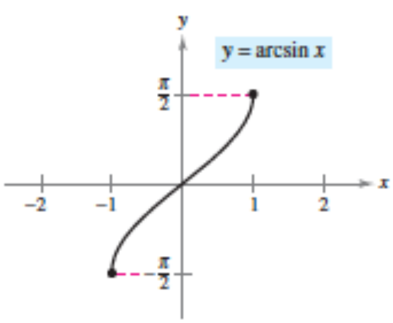
y = Sec x, (0, π) not π/2



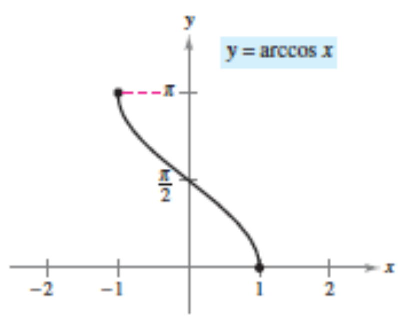
y = Csc x, (-π/2, π/2) not 0



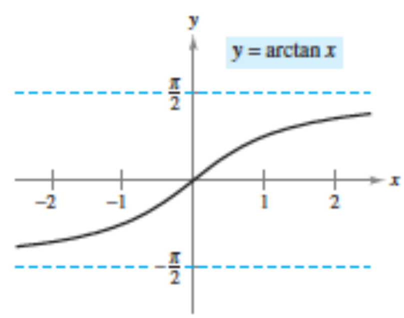
The Inverse for a Trigonometric Function is indicated by Adding the prefix "arc" or a superscript of -1 after the function. For instance, the Inverse of y = Sin x, is written as y = arcsin x or y = $\sin^{-1}$x or y = asin x.
Graphs of Inverse Trigonometric Functions



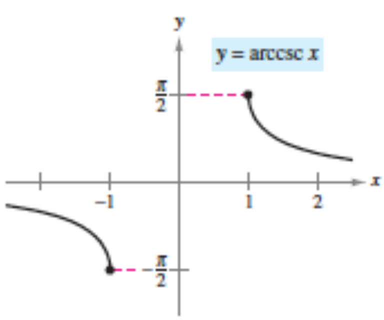
Domain: [−1, 1]
Range: [−π/2, π/2]



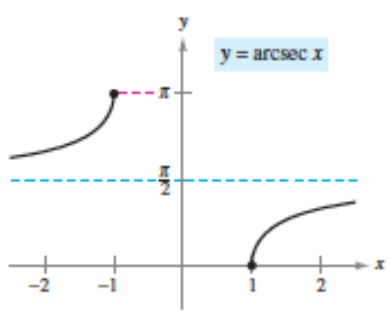
Domain: [−1, 1]
Range: [0, π]



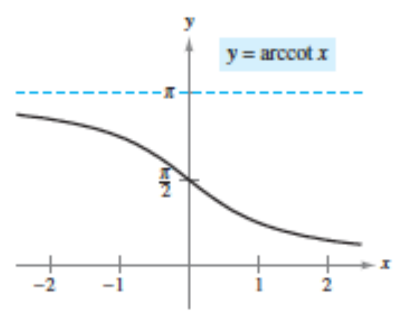
Domain: (−∞, ∞)
Range: (−π/2, π/2)



Domain: (−∞, −1] ∪ [1, ∞)
Range: [−π/2, 0) ∪ (0, π/2]



Domain: (−∞, −1] ∪ [1, ∞)
Range: [0, π/2) ∪ (π/2, π]



Domain: (−∞, ∞)
Range: (0, π)

Derivatives of Inverse Trig Functions:

1. $y = \arcsin x$ $\sin y = x \rightarrow \cos y = \sqrt{1-x^2}$ $\cos y \bullet \frac{dy}{dx} = 1$ $\frac{dy}{dx} = \frac{1}{\cos y} = \boxed{\frac{1}{\sqrt{1 - x^2}}}$ $\frac{d}{dx}(\arcsin u) = \boxed{\frac{1}{\sqrt{1 - u^2}}\frac{du}{dx}}$	2. $y = \arccos x$ $\cos y = x \rightarrow \sin y = \sqrt{1-x^2}$ $-\sin y \bullet \frac{dy}{dx} = 1$ $\frac{dy}{dx} = -\frac{1}{\sin y} = \boxed{-\frac{1}{\sqrt{1 - x^2}}}$ $\frac{d}{dx}(\arccos u) = \boxed{-\frac{1}{\sqrt{1 - u^2}}\frac{du}{dx}}$
3. $y = \arctan x$ $\tan y = x \rightarrow \sec y = \sqrt{x^2 + 1}$ $\sec^2 y \bullet \frac{dy}{dx} = 1$ $\frac{dy}{dx} = \frac{1}{\sec^2 y} = \boxed{\frac{1}{1 + x^2}}$ $\frac{d}{dx}(\arctan u) = \boxed{\frac{1}{1 + u^2}\frac{du}{dx}}$	4. $y = \text{arccot } x$ $\cot y = x \rightarrow \csc y = \sqrt{1 + x^2}$ $-\csc y \bullet \frac{dy}{dx} = 1$ $\frac{dy}{dx} = -\frac{1}{\csc y} = \boxed{-\frac{1}{1 + x^2}}$ $\frac{d}{dx}(\text{arccot } u) = \boxed{-\frac{1}{1 + u^2}\frac{du}{dx}}$
5. $y = \text{arcsec } x$ $\sec y = x \rightarrow \tan y = \sqrt{x^2 - 1}$ $\sec y \tan y \bullet \frac{dy}{dx} = 1$ $\frac{dy}{dx} = \frac{1}{\sec y \tan y} = \boxed{\frac{1}{\lvert x \rvert\sqrt{x^2 - 1}}}$ $\frac{d}{dx}(\text{arcsec } u) = \boxed{\frac{1}{\lvert u \rvert\sqrt{u^2 - 1}}\frac{du}{dx}}$	6. $y = \text{arccsc } x$ $\csc y = x \rightarrow \cot y = \sqrt{x^2 - 1}$ $-\csc y \cot y \bullet \frac{dy}{dx} = 1$ $\frac{dy}{dx} = -\frac{1}{\csc y \cot y} = \boxed{-\frac{1}{\lvert x \rvert\sqrt{x^2 - 1}}}$ $\frac{d}{dx}(\text{arccsc } u) = \boxed{-\frac{1}{\lvert u \rvert\sqrt{u^2 - 1}}\frac{du}{dx}}$

Alternate Versions of 1-6

1. $y = \sin^{-1}x \rightarrow x = \sin y \rightarrow 1 = \cos y \frac{dy}{dx} \rightarrow \frac{dy}{dx} = \sec y$

If x = sin y, then there is a right triangle with opp = x, hyp = 1, adj = $\sqrt{1 - x^2}$

$\sec y = \frac{1}{\sqrt{1 - x^2}} \rightarrow \boxed{\frac{d}{dx}[\sin^{-1}x] = \frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1 - x^2}}}$

2. $y = \cos^{-1}x \rightarrow x = \cos y \rightarrow 1 = -\sin y \frac{dy}{dx} \rightarrow \frac{dy}{dx} = -\csc y$

If x = cos y, then there is a right triangle with adj = x, hyp = 1, opp = $\sqrt{1 - x^2}$

$-\csc y = -\frac{1}{\sqrt{1 - x^2}} \rightarrow \boxed{\frac{d}{dx}[\cos^{-1}x] = \frac{d}{dx}[\arccos x] = -\frac{1}{\sqrt{1 - x^2}}}$

2. $y = \tan^{-1}x \rightarrow x = \tan y \rightarrow 1 = \sec^2 y \frac{dy}{dx} \rightarrow \frac{dy}{dx} = \cos^2 y$

If x = tan y, then there is a right triangle with opp = x, adj = 1, hyp = $\sqrt{x^2 + 1}$

$\cos^2 y = \frac{1}{x^2 + 1} \rightarrow \boxed{\frac{d}{dx}[\tan^{-1}x] = \frac{d}{dx}[\arctan x] = \frac{1}{x^2 + 1}}$

4. $y = \cot^{-1}x \rightarrow x = \cot y \rightarrow 1 = -\csc^2 y \frac{dy}{dx} \rightarrow \frac{dy}{dx} = -\sin^2 y$

If x = cot y, then there is a right triangle with adj = x, opp = 1, hyp = $\sqrt{x^2 + 1}$

$-\sin^2 y = -\frac{1}{\sqrt{x^2 + 1}} \rightarrow \boxed{\frac{d}{dx}[\cot^{-1}y] = \frac{d}{dx}[\text{arccot } x] = -\frac{1}{x^2 + 1}}$

5. $y = \sec^{-1}x \rightarrow x = \sec y \rightarrow 1 = \tan y \sec y \frac{dy}{dx} \rightarrow \frac{dy}{dx} = \cot y \cos x$

If x = sec y, then there is a right triangle with hyp = x, adj = 1, opp = $\sqrt{x^2 - 1}$

$\cot y \cos y = \frac{1}{|x|\sqrt{x^2 - 1}} \rightarrow \boxed{\frac{d}{dx}[\sec^{-1}x] = \frac{d}{dx}[\text{arcsec } x] = \frac{1}{|x|\sqrt{x^2 - 1}}}$

6. $y = \csc^{-1}x \rightarrow x = \csc y \rightarrow 1 = -\cot y \csc y \frac{dy}{dx} \rightarrow \frac{dy}{dx} = -\tan y \sin y$

If x = csc y, then there is a right triangle with hyp = x, opp = 1, adj = $\sqrt{x^2 - 1}$

$-\tan y \sin y = -\frac{1}{|x|\sqrt{x^2 - 1}} \rightarrow \boxed{\frac{d}{dx}[\csc^{-1}x] = \frac{d}{dx}[\text{arcscs } x] = -\frac{1}{|x|\sqrt{x^2 - 1}}}$

AP Calculus BC 1 Assignment 144 Tuesday, December 8, 2015 Hour Name

NO CALCULATORS

1.	$\frac{d}{dx}\arccos(2x^2) =$
2.	$\frac{d}{dx}\arctan\left(\frac{1}{2x}\right) =$
3.	$\frac{d}{dx}3^{5x} =$
4.	$\int 3^{5x}\,dx =$
5.	$\frac{d}{dx}\left[\log_5\left(7x^2\right)\right] =$
6.	$\int x\log_7\left(2x^2+3\right)dx =$
7.	$\frac{d}{dx}\text{arcsec } x$
8.	$\frac{d}{dx}\left[\log_5\left(7x^2\right)\right]$