

## Example 1:

Write a polynomial function of least degree with rational coefficients, a leading coefficient of 1, and the zeros are: 3 and  $2 + \sqrt{5}$ .

*Solution:*

Since  $2 + \sqrt{5}$  is a zero, we know that  $2 - \sqrt{5}$  is also a zero. Moreover, knowing the zeros, gives us access to the factors of  $f(x)$ . Remember that if  $c$  is a zero, then  $(x - c)$  is a factor.

$$\begin{aligned} \text{Multiplying the factors: } (x-3)(x-(2-\sqrt{5}))(x-(2+\sqrt{5})) &= (x-3)([x-2] + \sqrt{5})([x-2] - \sqrt{5}) \\ &= (x-3)(x^2 - 4x + 4 - 5) = (x-3)(x^2 - 4x - 1) = x^3 - 4x^2 - x - 3x^2 + 12x + 3 = \boxed{x^3 - 7x^2 + 11x + 3} \end{aligned}$$

## Example 2:

Given the two functions:  $f(x) = 5x^3 - 8x^2 - 35x - 5$  and  $g(x) = 5x^2 - x^3 + 6x + 7$ .

Use a graphing calculator to find the solutions for  $f(x) = g(x)$ .

*Solution:*

Since  $f(x) = g(x)$ , then  $f(x) - g(x) = 0$ . Therefore:  $(5x^3 - 8x^2 - 35x - 5) - (5x^2 - x^3 + 6x + 7) = 0$ .

Combining like terms we get:  $6x^3 - 13x^2 - 41x - 12 = 0$ . Enter this into the calculator equation editor.

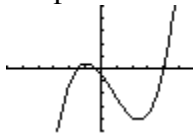
## Equation

```
Plot1 Plot2 Plot3
Y1=6X^3-13X^2-41
X=-12
Y2=
Y3=
Y4=
Y5=
Y6=
```

## Window

```
WINDOW
Xmin=-6
Xmax=6
Xscl=1
Ymin=-125
Ymax=125
Yscl=25
Xres=1
```

## Graph

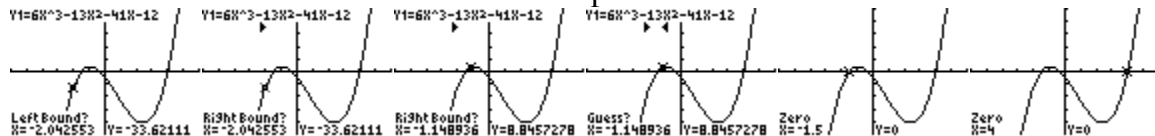


Now use the calculate function to find the zeros (the x-intercepts)

## Pick zero

```
Calculate
1:value
2:zero
3:minimum
4:maximum
5:intersect
6:dy/dx
7:∫f(x)dx
```

Press Enter on the each side of each x-intercept



We get  $x = -1.5$ . Repeat the process for the other two x-intercepts and we will get  $x = -0.333$  and  $x = 4$

We can verify these 3 solutions algebraically.

$$\begin{array}{r|rrrr} 4 & 6 & -13 & -41 & -12 \\ & & 24 & 44 & 12 \\ \hline & 6 & 11 & 3 & 0 \end{array}$$

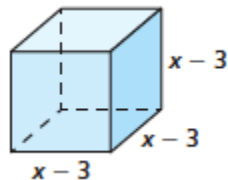
$$(x-4)(6x^2 - 11x + 3) \rightarrow (x-4)(6x-9)(6x-2) \rightarrow (x-4)(2x-3)(3x-1) = 0$$

$$x = 4, \frac{3}{2}, \frac{1}{3}$$

Write a polynomial function  $f$  of least degree that has a leading coefficient of 1 and the given zeros.

1.  $-2, 3, 6$
2.  $-4, -2, 5$
3.  $-2, 1 + \sqrt{7}$
4.  $4, 6 - \sqrt{7}$
5.  $-6, 0, 3 - \sqrt{5}$
6.  $0, 5, -5 + \sqrt{8}$

7. The volume of the cube shown below is 8 cubic centimeters.



- a. Write a polynomial equation that you can use to find the value of  $x$ .
- b. Identify the possible rational solutions of the equations in part (a).
- c. Use synthetic division to find all rational solutions of the equation.
- d. Find the dimensions of the cube.

Solve  $f(x) = g(x)$  by graphing and check by algebraic methods.

8.  $f(x) = x^3 + x^2 - x - 1$ ;  $g(x) = -x + 1$
9.  $f(x) = x^4 - 5x^3 + 2x^2 + 8$ ;  $g(x) = -x^2 + 6x - 5$