Algebra 2
143

The Rational Root Theorem

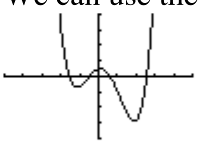
The solutions of the equation $64x^3 + 152x^2 - 62x - 105 = 0$ are $-\frac{5}{2}, -\frac{3}{4}, \& \frac{7}{8}$

Notice that the numerators (5, 3, & 7) are factors of 105 and the denominators (2, 4, & 8) are factors of 64. Generally if $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1 x + a_0$ has integer coefficients, then every rational solution of f(x) has the following form: $\frac{p}{q} = \frac{\text{factor of } a_0}{\text{factor of } a_n}$. NOTE: This Theorem DOES NOT give the solutions, only possible solutions.

Example 1:
$f(x) = 10x^4 - 11x^3 - 42x^2 + 7x + 12$ Find all real zeros.
Solution:

The Rational Candidates are: $x = \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{4}{1}, \pm\frac{6}{1}, \pm\frac{12}{1}, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{5}, \pm\frac{2}{5}, \pm\frac{3}{5}, \pm\frac{4}{5}, \pm\frac{6}{5}, \pm\frac{12}{5}, \pm\frac{1}{10}, \pm\frac{3}{10}$

We can use the graphing calculator to get an idea of the best choices.



The x-intercepts (zeros) are: in [-2, -1], in [-1, 0], in [0, 1], and in [3, 4].

Try $-\frac{3}{2}$ or $-1\frac{1}{2}$

-1 1/2	10	-11	-42	7	12
		-15	39	4 1/2	-17 1/4
	10	-26	-3	11 1/2	-5 1/4

$-\frac{3}{2}$ is not a zero

Try $-\frac{1}{2}$

- 1/2	10	-11	-42	7	12
		-5	8	17	-12
	10	-16	-34	24	0

$-\frac{1}{2}$ is a zero

So far we have $f(x) = (x + ½)(10x^3 - 16x^2 - 34x + 24)$
For $10x^3 - 16x^2 - 34x + 24$, we will try to find zeros.

Try $\frac{3}{5}$

3/5	10	-16	-34	24
		6	-6	-24
	10	-10	-40	0

$\frac{3}{5}$ is a zero.

So now we have: $f(x) = (x + ½)(x - 3/5)(10x^2 - 10x - 40)$
In the last parentheses we can divide by 2 but multiply the first parentheses by 2
Also in the last parentheses we can divide by 5 but multiply the 2nd parentheses by 5
$(2x + 1)(5x - 3)(x^2 - x - 4)$ Since $x^2 - x - 8$ cannot be factored, so we must use the quadratic formula to find the remaining zeros.

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-4)}}{2} = \frac{1 \pm \sqrt{1+16}}{2} = \frac{1 \pm \sqrt{17}}{2}$$

Now, we have all the zeros: $\boxed{-\frac{1}{2}, \frac{3}{5}, \frac{1+\sqrt{17}}{2}, \& \frac{1-\sqrt{17}}{2}}$

Example 2:
$f(x) = x^3 - 8x^2 + 11x + 20$ Find all real roots (zeros).
Solution:

The Rational Candidates are: $x = \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{4}{1}, \pm\frac{5}{1}, \pm, \frac{10}{1}, \pm\frac{20}{1} = \pm 1, \pm 2, \pm 4, \pm 5, \pm, 10, \pm 20$

Try 1

1	1	-8	11	20
		1	-7	4
	1	-7	4	24

1 did not give a zero.

Try -1

-1	1	-8	11	20
		-1	9	-20
	1	-9	20	0

-1 gave a zero.

So far we have the factorization: $f(x) = (x + 1)(x^2 - 9x + 20)$
To finish the factorization: $f(x) = (x + 1)(x + 4)(x - 5)$
The zeros are: $\boxed{-1, -4, \& -5}$

The Irrational Conjugates Theorem:
If f(x) is a polynomial with rational coefficients and $a + \sqrt{b}$ is a zero, where $\sqrt{b}$ is irrational, then $a - \sqrt{b}$ must also be a zero of f(x).

Example 3:
Solve: $x^3 - 2x^2 = 16x - 32$
Solution:
$x^3 - 2x^2 - 16x + 32 = 0$ Group the expression: $(x^3 - 2x^2) - (16x - 32) = x^2(x - 2) - 16(x - 2)$
$= (x - 2)(x^2 - 16) = (x - 2)(x + 4)(x - 4)$. Therefore the zeros are: $\boxed{2, -4, \& 4}$

Algebra 2 Assignment 143 Monday December 7 2015 Hour Name

Solve the following equations:

1. $x^3 - 10x^2 + 19x + 30 = 0$ Try -1

2. $x^3 + 4x^2 - 11x - 30 = 0$ Try -2

3. $x^3 - 6x^2 - 7x + 60 = 0$ Try =3

4. $x^3 - 16x^2 + 55x + 72 = 0$ Try -1

5. $2x^3 - 3x^2 - 50x - 24 = 0$ Try -4

6. $3x^3 + x^2 - 38x + 24 = 0$ Try 3

Find all the real zeros of the function:

7. $f(x) = x^3 - 2x^2 - 23x + 60$ Try -5

8. $g(x) = x^3 - 28x - 48$ Try 6

9. $h(x) = x^3 + 10x^2 + 31x + 30$ Try -2

10. $f(x) = x^3 - 14x^2 + 55x - 42$ Try 1

11. $p(x) = 2x^3 - x^2 - 27x + 36$ Try 3

12. $g(x) = 3x^3 - 25x^2 + 58x - 40$ Try 5