

Arc Length of a curve. The following applies for a differentiable curve that does not cross itself on the given interval.



A representative “CHUNK” of length is a hypotenuse which has length $\sqrt{(dx)^2 + (dy)^2}$

The length of a curve on the interval $[a,b] = \int_a^b \sqrt{(dx)^2 + (dy)^2}$, but this form is not ready for integration.

Therefore, we make an alteration of the form.

$$\int_a^b \sqrt{\left(\frac{dx}{dx}\right)^2 + \left(\frac{dy}{dx}\right)^2} dx = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

Example:

Set up the Integral, then use the Calculator to find the arc length of $y = 2x^2 + \sin 3x$ on $[-2, 5]$.

Solution:

$$y' = 4x + 3 \cos 3x$$

$$L = \int_{-2}^5 \sqrt{1 + (4x + 3 \cos 3x)^2} dx = \boxed{61.0201}$$

Find the Length of the Function y on the Given Interval. Show Set-Up on Paper.

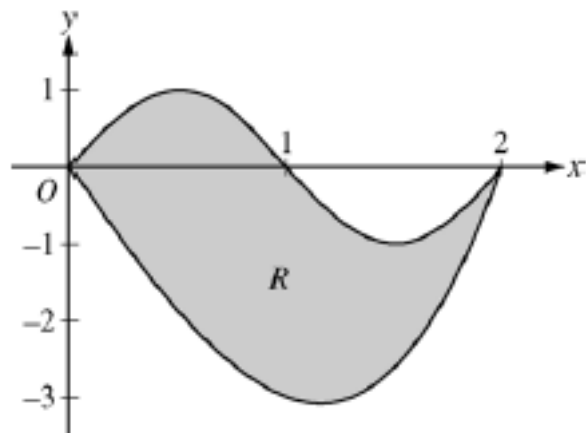
Use Calculator to Compute.

1. $y = \frac{2}{3} x^{3/2} + 1, \quad [0, 1]$

2. $y = \frac{3}{2} x^{2/3} + 4, \quad [1, 27]$

3. $y = \ln(\sin x), \quad \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

4. $x = \sqrt{36 - y^2}, \quad [0, 3]$



5. Region R is bounded by $f(x) = \sin(\pi x)$ and $g(x) = x^3 - 4x$

a. Find the area of R.

b. The line $y = -2$ splits region R into two parts. Find the area of the part of R that is below $y = -2$.

c. Region R is the base of a solid whose cross-sections perpendicular to the x-axis are squares. Find the volume of this solid.

d. Region R models the surface of a body of water. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of body of water.