

We can often factor a polynomial if we can find a zero of the polynomial. This will allow us to work with a reduced degree polynomial.

Example 1:

$$f(x) = 3x^3 + 5x^2 - 34x - 24$$

None of our previous methods apply to factoring this above polynomial. We will make a guess to find a zero using synthetic substitution.

Try 1

$$\begin{array}{r|rrrr} 1 & 3 & 5 & -34 & -24 \\ & & 3 & 8 & -26 \\ \hline & 3 & 8 & -26 & -50 \end{array}$$

1 did not give a zero.

Try 2

$$\begin{array}{r|rrrr} 2 & 3 & 5 & -34 & -24 \\ & & 6 & 22 & -24 \\ \hline & 3 & 11 & -12 & -48 \end{array}$$

2 did not give a zero.

Try 3

$$\begin{array}{r|rrrr} 3 & 3 & 5 & -34 & -24 \\ & & 9 & 42 & 24 \\ \hline & 3 & 14 & 8 & 0 \end{array}$$

3 gave a zero.

Since 3 is a zero, then $x - 3$ is a factor. Also: $3x^2 + 14x + 8$ is also a factor.

$$\text{So far, } f(x) = (x - 3)(3x^2 + 14x + 8)$$

At this point we will factor $3x^2 + 14x + 8$ by any method you choose.

$$3x^2 + 14x + 8 = (3x + 2)(x + 4)$$

$$\text{The complete factorization of } f(x) = \boxed{(x - 3)(3x + 2)(x + 4)}.$$

Tip: If the coefficients add to zero, then 1 is a zero. If the sum of the even degree coefficients equals the sum of the odd degree coefficients, then -1 is a zero.

Example 2:

$$f(x) = 8x^3 + 18x^2 + 7x - 3$$

The sum of the odd degree coefficients = $8 + 7 = 15$. The sum of the even degree coefficients = $18 - 3 = 15$.

Try -1

$$\begin{array}{r|rrrr} -1 & 8 & 18 & 7 & -3 \\ & & -8 & -10 & 3 \\ \hline & 8 & 10 & -3 & 0 \end{array}$$

-1 gives a zero.

Since -1 is a zero, then $x + 1$ is a factor. Also: $8x^2 + 10x - 3$ is also a factor.

$$\text{So far, } f(x) = (x + 1)(8x^2 + 10x - 3)$$

At this point we will factor $8x^2 + 10x - 3$ into $(4x - 1)(2x + 3)$

$$\text{The complete factorization of } f(x) = \boxed{(x + 1)(4x - 1)(2x + 3)}.$$

Factor the following, using the given factor indicating a zero.

1. $g(x) = x^3 - x^2 - 20x ; x + 4$

2. $f(x) = x^3 - 5x^2 - 9x + 45 ; x - 5$

3. $f(x) = x^4 - 6x^3 - 8x + 48 ; x - 6$

4. $s(x) = x^4 + 4x^3 - 64x - 256 ; x + 4$

5. $r(x) = x^3 - 37x + 84 ; x + 7$

6. $h(x) = x^3 - x^2 - 24x - 36 ; x + 2$

Factor the following.

7. $a^6 + a^5 - 30a^4$

8. $8m^3 - 343$

9. $z^3 - 7z^2 - 9z + 63$

10. $2p^8 - 12p^5 + 16p^2$

11. $64r^3 + 729$

12. $5x^5 - 10x^4 - 40x^3$

13. $16n^4 - 1$

14. $9k^3 - 24k^2 + 3k - 8$