AP Calculus BC
140

Finding Approximate Area using n-Trapezoids

Before we begin, we need to recall the formula for the sum of a geometric series:

$$\sum_{k=0}^{n} ar^k = \frac{a\left(r^{n+1}-1\right)}{r-1} \quad \text{or} \quad \sum_{k=1}^{n} r^k = \frac{r\left(r^n-1\right)}{r-1}$$

Find the trapezoidal approximation for $\int_0^1 2^x\,dx$, using n trapezoids of equal width.

Each trapezoid has width $\frac{1}{n}$.

Consider the kth trapezoid:

The bottom right corner is at x = $\frac{k}{n}$.

The bottom left corner is at x = $\frac{k-1}{n}$.

The height on the right side is $f\left(\frac{k}{n}\right) = 2^{k/n}$

The height of the left side is $f\left(\frac{k-1}{n}\right) = 2^{(k-1)/n}$

The area of the kth Trapezoid is: $\frac{1}{n} \bullet \frac{2^{k/n} + 2^{(k-1)/n}}{2} = \frac{1}{2n} \bullet \left[2^{k/n} + 2^{(k-1)/n}\right]$

$= \frac{1}{2n} \bullet \left[2^{k/n}\left(1+2^{-1/n}\right)\right] = \frac{1}{2n}\left(1+2^{-1/n}\right) \bullet \left(2^{1/n}\right)^k$

The sum of the areas of all the trapezoids is: $\sum_{k=1}^{n} \frac{1}{2n}\left(1+2^{-1/n}\right) \bullet \left(2^{1/n}\right)^k$

$= \frac{1}{2n}\left(1+2^{-1/n}\right) \bullet \sum_{k=1}^{n} \left(2^{1/n}\right)^k$ Recall that $\sum_{k=1}^{n} \left(2^{1/n}\right)^k$ is a geometric series.

$= \frac{1}{2n}\left(1+2^{-1/n}\right) \bullet \frac{2^{1/n}\left(2^{n/n}-1\right)}{2^{1/n}-1} = \frac{1}{2n}\left(1+2^{-1/n}\right) \bullet \frac{2^{1/n}}{2^{1/n}-1} = \frac{1}{2n} \bullet \frac{2^{1/n}+1}{2^{1/n}-1}$

Using a calculator to find the integral, we get 1.442695040889

Using a calculator, evaluate when n = 10, we get 1.4432726172912

Using a calculator, evaluate when n = 100, we get 1.4427008171148

Using a calculator, evaluate when n = 1000, we get 1.4426950986109

Let f(x) be continuous on [a, b]. Let n = the number of equal sub-intervals between x = a and x = b

$$\int_a^b f(x)\,dx \approx \frac{b-a}{n}\left[\frac{f(a)+f(x_1)}{2}+\frac{f(x_1)+f(x_2)}{2}+\frac{f(x_2)+f(x_3)}{2}+\cdots+\frac{f(x_{n-2})+f(x_{n-1})}{2}+\frac{f(x_{n-1})+f(b)}{2}\right]$$

$$= \frac{b-a}{2n}\left[f(a)+f(x_1)+f(x_1)+f(x_2)+f(x_2)+f(x_3)+f(x_3)+\cdots+f(x_{n-1})+f(x_{n-1})+f(b)\right]$$

$$= \frac{b-a}{2n}\left[f(a)+2f(x_1)+2f(x_2)+2(f(x_3)+\cdots+2f(x_{n-1})+f(b)\right]$$

The Trapezoidal Rule: Let f be continuous on [a,b]. The Trapezoidal Rule for approximating

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}\left[f(x_0)+2f(x_1)+2f(x_2)+2(f(x_3)+\cdots+2f(x_{n-1})+f(x_n)\right]$$

Often, you will be given a table or a graph to represent a function and asked to approximate a definite integral.

x	3	5	7	10	13	14	15
f(x)	8	10	13	12	15	16	14

1. Use A Right Sided Riemann Sum: $\int_3^{10} f(x)\,dx$

   2(10) + 2(13) + 3(12) = 82

2. Use a Left Sided Riemann Sum: $\int_7^{14} f(x)\,dx$

   3(13) + 3((12) + 1(15) = 90

3. Use a Trapezoidal Sum: $\int_5^{14} f(x)\,dx$

   2(10+13)/2 + 3(13+12)/2 + 3(12+15)/2 + 1(15+16)/2 = 116.5

4. Use a Center Valued Riemann Sum with 3 intervals: $\int_3^{15} f(x)\,dx$

   4(10) + 6(12) + 2(16) = 144

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Show each set-up process, but let your calculator do the arithmetic for accuracy purposes.

x	3	4	6	9	11
f(x)	2	8	6	10	4

The table above gives selected values of the continuous function f(x).

Find the approximate value of $\int_{3}^{11} f(x)\,dx$ using:

1. The Trapezoidal Approximation

2. The Left side Riemann Approximation

3. The Right side Riemann Approximation

4. The Center Riemann Approximation

Exer. 5-7: Use trapezoids to find the numerical approximation

5. $\int_4^9 \sqrt{x}\, dx$, n = 8

6. $\int_0^2 \sqrt{1+x^3}\, dx$, n = 4

7. $\int_0^{\pi/4} x \tan x\, dx$, n = 4