

Divide  $2x^4 + 3x^3 + 5x - 1$  by  $x^2 + 3x + 2$

$\begin{array}{r} 2x^2 - 3x + 5 \leftarrow \text{quotient} \\ x^2 + 3x + 2 \overline{) 2x^4 + 3x^3 + 0x^2 + 5x - 1} \\ \underline{2x^4 + 6x^3 + 4x^2} \phantom{- 1} \\ -3x^3 - 4x^2 + 5x \phantom{- 1} \\ \underline{-3x^3 - 9x^2 - 6x} \phantom{- 1} \\ 5x^2 + 11x - 1 \phantom{- 1} \\ \underline{5x^2 + 15x + 10} \\ -4x - 11 \leftarrow \text{remainder} \end{array}$	<p>Multiply divisor by <math>\frac{2x^4}{x^2} = 2x^2</math>.</p> <p>Subtract. Bring down next term.</p> <p>Multiply divisor by <math>\frac{-3x^3}{x^2} = -3x</math>.</p> <p>Subtract. Bring down next term.</p> <p>Multiply divisor by <math>\frac{5x^2}{x^2} = 5</math>.</p>
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The Answer:  $\frac{2x^4 + 3x^3 + 5x - 1}{x^2 + 3x + 2} = \boxed{2x^2 - 3x + 5 + \frac{-4x - 11}{x^2 + 3x + 2}}$

Try These Divisions. The Answers are there for you to check:

1.  $(x^3 - x^2 - 2x + 8) \div (x - 1) \qquad x^2 - 2 + \frac{6}{x - 1}$

2.  $(x^4 + 2x^2 - x + 5) \div (x^2 - x + 1) \qquad x^2 + x + 2 + \frac{3}{x^2 - x + 1}$

Synthetic Division can be used when the Divisor is in the form of  $(x - a)$ . We use  $k = -a$ .

Divide  $-x^3 + 4x^2 + 9$  by  $x - 3$

Write the coefficients of the dividend in order of descending exponents.

Include a "0" for the missing x-term. Because the divisor is  $x - 3$ , use  $k = 3$ .

Write the k-value to the left of the vertical bar.

$k\text{-value} \rightarrow 3 \mid -1 \quad 4 \quad 0 \quad 9 \leftarrow \text{coefficients of } -x^3 + 4x^2 + 9$

Bring down the leading coefficient. Multiply the leading coefficient by the k-value.

Write the product under the second coefficient. Add.

$3 \mid -1 \quad 4 \quad 0 \quad 9$   
 $\phantom{3} \mid -1 \quad 1 \phantom{0} \phantom{0} \phantom{0}$

Multiply the previous sum by the k-value. Write the product under the third coefficient. Add. Repeat this process for the remaining coefficient. The first three numbers in the bottom row are the coefficients of the quotient, and the last number is the remainder.

$3 \mid -1 \quad 4 \quad 0 \quad 9$   
 $\phantom{3} \mid -1 \quad 1 \quad 3 \quad 9$   
 $\phantom{3} \mid -1 \quad 1 \quad 3 \quad 18 \leftarrow \text{remainder}$

$\frac{-x^3 + 4x^2 + 9}{x - 3} = -x^2 + x + 3 + \frac{18}{x - 3}$

Divide  $(3x^3 - 2x^2 + 2x - 5)$  by  $x - 1$

We can use synthetic division because the divisor is  $x + 1$ , where  $k = -1$ .

$$\begin{array}{r|rrrr} -1 & 3 & -2 & 2 & -5 \\ & & -3 & 5 & -7 \\ \hline & 3 & -5 & 7 & -12 \end{array}$$

$$\boxed{\frac{3x^3 - 2x^2 + 2x - 5}{x + 1} = 3x^2 - 5x + 7 - \frac{12}{x + 1}}$$

The Remainder Theorem

In a previous example we divided  $(3x^3 - 2x^2 + 2x - 5)$  by  $x - 1$ , using Synthetic Division.

The last number (Remainder) under the line was -12. If  $f(x) = (3x^3 - 2x^2 + 2x - 5)$  and we want to find  $f(-1)$ , we can compute:  $f(-1) = (3(-1)^3 - 2(-1)^2 + 2(-1) - 5) = -3 - 2 - 2 - 5 = -12$

So now we can Synthetically Substitute a number to evaluate a function, using the synthetic division process.

$f(x) = 5x^3 - x^2 + 13x + 29$  Find  $f(-4)$  using the Synthetic Process.

$$\begin{array}{r|rrrr} -4 & 5 & -1 & 13 & 29 \\ & & -20 & 84 & -388 \\ \hline & 5 & -21 & 97 & -359 \end{array}$$

Therefore:  $f(-4) = \boxed{-359}$

Divide the following.

1.  $(x^2 + x - 17) \div (x - 4)$

2.  $(x^3 + x^2 + x + 2) \div (x^2 + 1)$

3.  $(5x^4 - 2x^3 - 7x^2 - 39) \div (x^2 + 2x - 4)$

4.  $(4x^4 + 5x - 4) \div (x^2 - 3x - 2)$

5.  $(x^4 + 4x^3 + 16x - 35) \div (x + 5)$

6.  $(x^2 + x - 3) \div (x - 2)$

Use Synthetic Division (Synthetic Substitution) to evaluate the following for the given value of  $x$ .

7.  $f(x) = -x^2 - 8x + 30; x = -1$

8.  $f(x) = 3x^2 + 2x - 20; x = 3$

9.  $f(x) = x^3 - 2x^2 + 4x + 3; x = 2$

10.  $f(x) = x^3 + x^2 - 3x + 9; x = -4$

11.  $f(x) = x^4 + 6x^2 - 7x + 1; x = 3$

12.  $f(x) = -x^4 - x^3 - 2; x = 5$