Change of Variables:

$\int x\sqrt{2x-1}\,dx$

u = 2x - 1 □ x = $\frac{u+1}{2}$ □ dx = $\frac{1}{2}du$

$\int\left(\frac{u+1}{2}\right)u^{1/2}\,\frac{1}{2}du$

$= \frac{1}{4}\int\left(u^{3/2}+u^{1/2}\right)du$

$= \frac{1}{4}\left(\frac{2}{5}u^{5/2}+\frac{2}{3}u^{3/2}\right)+C$

$= \frac{1}{10}u^{5/2}+\frac{1}{6}u^{3/2}+C$

$= \boxed{\frac{1}{10}(2x-1)^{5/2}+\frac{1}{6}(2x-1)^{3/2}+C}$.

$\int_1^5 \frac{x}{\sqrt{2x-1}}\,dx$

u = 2x – 1 $\rightarrow$ x = $\frac{1}{2}(u+1))$ $\rightarrow$ $dx = \frac{1}{2}du$

When x = 1, u = 1 & When x = 5, u = 9

$\int_1^9 \frac{1}{2}\bullet\frac{u+1}{u^{1/2}}\bullet\frac{1}{2}\,du$

$= \frac{1}{4}\int_1^9 u^{-1/2}(u+1)du$

$= \frac{1}{4}\int_1^9\left(u^{1/2}+u^{-1/2}\right)du$

$= \frac{1}{4}\left[\frac{2}{3}u^{3/2}+2u^{1/2}\right]_1^9$

$= \left[(18=6)-\left(\frac{2}{3}+2\right)\right]$

$= \frac{1}{4}\left(24-2-\frac{2}{3}\right)$

$= \frac{1}{4}\left(\frac{64}{3}\right)$

$= \boxed{\frac{16}{3}}$.

Alternate Choice

$$\int_1^5 \frac{x}{\sqrt{2x-1}}\,dx$$

u = $\sqrt{2x-1}$ $\rightarrow$ $u^2 = 2x - 1$ $\rightarrow$ x = $\frac{1}{2}\left(u^2+1\right)$ $\rightarrow$ dx = u du

When x = 1, u = 1 When x = 5, u = 3

$$\int_1^3 u^{-1} \bullet \frac{1}{2}\left(u^2+1\right)u\,du$$

$$= \frac{1}{2}\int_1^3 \left(u^2+1\right)du$$

$$= \frac{1}{2}\left[\frac{1}{3}u^3+u\right]_1^3$$

$$= \frac{1}{2}\left[(9+3)-\left(\frac{1}{3}+1\right)\right]$$

$$= \frac{1}{2}\left(11-\frac{1}{3}\right)$$

$$= \frac{1}{2}\left(\frac{32}{3}\right)$$

$$= \boxed{\frac{16}{3}}.$$

AP Calculus BC 1 Assignment 138 Friday, November 20, 2015 Hour Name

1. $\int x\sqrt{x+2}\,dx$

2. $\int x\sqrt{2x+1}\,dx$

3. $\int x^2\sqrt{1-x}\,dx$

4. $\int (x+1)\sqrt{2-x}\,dx$

5. $\int \frac{x^2 - 1}{\sqrt{2x-1}}\, dx$

6. $\int_0^4 \frac{1}{\sqrt{2x+1}}\, dx$

7. $\int_0^2 \frac{x}{\sqrt{1+2x^2}}\, dx$

8. $\int_1^2 (x-1)\sqrt{2-x}\, dx$