

Several Examples of the expansion of  $(x + y)^n$  for  $n = 0, 1, 2, 3, 4, 5$  are:

$$(x + y)^0 = 1$$

$$(x + y)^1 = x + y$$

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

$$(x + y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$$

In each case:

1. The number of terms is  $n + 1$
2. The powers of  $x$  decrease by 1 on each successive term
3. The powers of  $y$  increase by 1 on each successive term
4. The coefficients are symmetric about the center term
5. For each term The sum of the powers of  $x$  and  $y$  are  $n$

Pascal's Triangle:

0								1																
1							1		1															
2						1		2		1														
3					1		3		3		1													
4				1		4		6		4		1												
5				1		5		10		10		5		1										
6				1		6		15		20		15		6		1								
7				1		7		21		35		35		21		7		1						
8				1		8		28		56		70		56		28		8		1				
9				1		9		36		84		126		126		84		36		9		1		
10				1		10		45		120		210		252		210		120		45		10		1

Pascal's Triangle can be used to find the coefficients of the above expansions.

Observe that each line starts and ends with 1 and all other values are the sums of the immediate above values.

Examples

1.  $(2x - 3y)^4$  Use the 4 line of Pascal's Triangle. The powers of  $2x$  descend and the powers of  $(-3y)$  ascend.  
 $= 1(2x)^4 + 4(2x)^3(-3y) + 6(2x)^2(-3y)^2 + 4(2x)(-3y)^3 + 1(-3y)^4$   
 $= 16x^4 + 4(8x^3)(-3y) + 6(4x^2)(9y^2) + 4(2x)(-27y^3) + 81y^4$   
 $= \boxed{16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4}$
2.  $(3x - 2)^3$  Use the 3 line of Pascal's Triangle.  
 $= 1(3x)^3 + 3(3x)^2(-2) + 3(3x)(-2)^2 + 1(-2)^3$   
 $= 27x^3 + 3(9x^2)(-2) + (9x)(4) - 8$   
 $= \boxed{27x^3 - 54x^2 + 36x - 8}$

Simplify the following.

1.  $(3x^2 + 4x - 1) + (-2x^2 - 3x + 2) =$

2.  $(7x^6 + 2x^5 - 3x^2 + 9x) + (5x^5 + 8x^3 - 6x^2 + 2x - 5)$

3.  $(3x^3 - 2x^2 + 4x - 8) - (5x^3 + 12x^2 - 3x - 4)$

4.  $(5x^6 - 2x^4 + 9x^3 + 2x - 4) - (7x^5 - 8x^4 + 2x - 11)$

5.  $(5x^2 - 4x + 6)(-2x + 3)$

6.  $(3x^2 + x - 2)(-4x^2 - 2x - 1)$

7.  $(4x^2 - 8x - 2)(x^4 + 3x^2 + 4x)$

8.  $(x - 3)(x + 2)(x + 4)$

9.  $(2x + 5)(x - 2)(3x + 4)$

10.  $(x - 9)(x + 9)$

11.  $(m + 6)^2$

12.  $(3c - 5)^2$

13.  $(2y - 5)(2y + 5)$

14.  $(2k + 6)^3$

15.  $(g + 2)^5$

16.  $(np - 1)^4$