

The Fundamental Theorem of Calculus Part 1 (FTC 1)

If a function f is continuous on the closed interval $[a, b]$ and F is an antiderivative of f on the interval (a, b) , then

$$\int_a^b f(x) dx = F(b) - F(a)$$

Mean Value Theorem for Integrals (Freeze-Thaw)

If f is continuous on the closed interval $[a, b]$, then there exists a number c in the closed interval $[a, b]$ such that

$$\int_a^b f(x) dx = f(c)(b - a)$$

The Definition of the **Average Value of a Function** on an Interval

If f is integrable on the closed interval $[a, b]$, then the

Average Value of f on the interval is $\frac{1}{b-a} \int_a^b f(x) dx$

The Fundamental Theorem of Calculus Part 2 (FTC 2)

If f is continuous on an open interval I containing a , then, for every x in the interval,

then $\forall x$ on the interval, $\frac{d}{dx} \left[\int_a^x f(t) dt \right] = f(x)$.

Let $f(t) = 3t^2 - 4t$

$$\begin{aligned} \frac{d}{dx} \int_3^{2x} f(t) dt &= \frac{d}{dx} \int_3^{2x} (3t^2 - 4t) dt = \frac{d}{dx} \left[t^3 - 2t^2 \right]_3^{2x} = \frac{d}{dx} \left[(8x^3 - 8x^2) - (27 - 18) \right] \\ &= \frac{d}{dx} [8x^3 - 8x^2 - 9] = 24x^2 - 16x = 2(12x^2 - 8x) = 2(3(2x)^2 - 4(2x)) = \boxed{2f(2x)} \end{aligned}$$

In general:

$$\boxed{\frac{d}{dx} \int_a^{g(x)} f(t) dt = f(g(x)) \cdot g'(x)}$$

1. $f(x) = \int_{22}^{\sin(5x)} \sqrt{1-t^2} dt$ Find $f'(x)$

2. Find the average value of $f(x) = 3 \sin(4x) + 7x^4$ on $[2, 6]$. Make your answer accurate to 3-decimal places.

3. $f(x) = \int_{5x}^9 \tan^2(7t^3) dt$ Find $f'(x)$

4. $\int \sec^2 2x \tan 2x dx$

5. $3xy^2 + 2x - 4y = \sin y$ Find $\frac{dy}{dx}$

6. Sand enters a processing plant at the rate of $E(t) = 90 + 45 \cos \frac{t^2}{18}$ tons/hour, where $t = 0$ at 8:00 AM. The plant's hours of operation are from 8:00 AM to 4:00 PM. Find $E'(5)$. Using proper units, interpret your answer in the context of the problem.

7. $g(x) = x^3 - 3x$ is defined on $(-3, 3)$ Show your Work and Find the x-coordinate(s) of the Maxima. Justify your answer.