A **monomial** is a number, a variable, or the product of a number and one or more variables with whole number exponents. A **polynomial** is a monomial or a sum of monomials. A **polynomial function** is a function of the form: $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0$, where $a_n \neq 0$, the exponents are all whole numbers, and the coefficients are all real numbers. For this function, a_n is the **leading coefficient**, n is the **degree**, and a_0 is the **constant term**.

A polynomial is in **standard form** when its terms when its terms are written in descending order of exponents from left to right.

Common Polynomial Functions			
Degree	**Type**	**Standard Form**	**Example**
0	Constant	$f(x) = a_0$	$f(x) = -14$
1	Linear	$f(x) = a_1x + a_0$	$f(x) = 5x - 7$
2	Quadratic	$f(x) = a_2x^2 + a_1x + a_0$	$f(x) = 2x^2 + x - 9$
3	Cubic	$f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$	$f(x) = x^3 - x^2 + 3x$
4	Quartic	$f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$	$f(x) = x^4 + 2x - 1$

Decide whether the functions are polynomials. If they are, do the following:

a. Write in standard form
b. State the degree, type, and leading coefficient.

1. $f(x) = -2x^3 + 5x + 8$
   A polynomial; already in standard form; degree 3; cubic; leading coefficient = -2

2. $g(x) = -0.8x^3 + \sqrt{2}x^4 - 12$
   A polynomial; $g(x) = \sqrt{2}x^4 - 0.8x^3 - 12$; degree 4; quartic; leading coefficient = -0.8

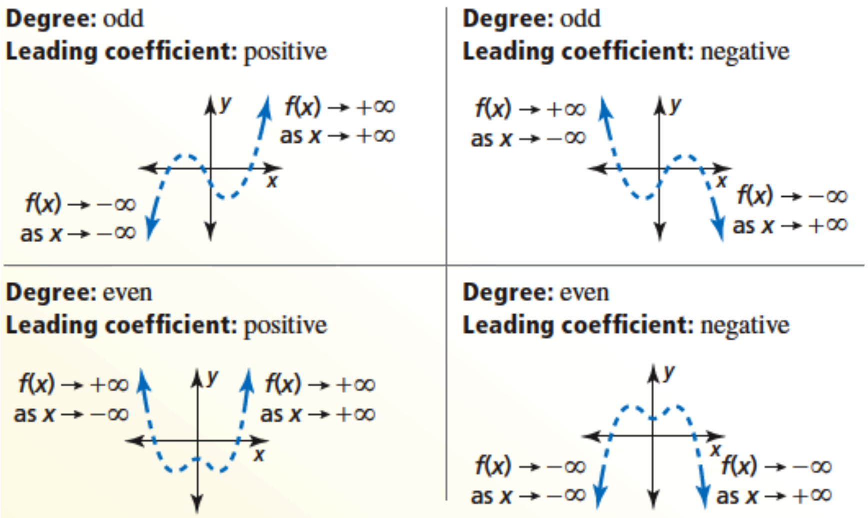
3. $h(x) = -x^2 + 7x^{-1} + 4x$
   Not a polynomial because the term $7x^{-1}$ has an exponent that is not a whole number

4. $k(x) = x^2 + 3^x$
   Not a polynomial because the term 3^x has a variable for the exponent

The **end behavior** of a function's graph is the behavior of the graph as x approaches positive infinity $(+\infty)$ or negative infinity $(-\infty)$. For the graph of a polynomial function, the end behavior is determined by the function's degree and the sign of its leading coefficient.

For Even Degree Polynomials, the End Behaviors are the same at both ends.
For Odd Degree Polynomials, the End Behaviors are opposite at each end.
If the Leading Coefficient is Positive, the end behavior on the right side is toward positive infinity.
If the Leading Coefficient is Negative, the end behavior on the right side is toward negative infinity.

End Behavior of Polynomial Functions



$f(x) \to +\infty$ as $x \to +\infty$ is read, "$f(x)$ approaches positive infinity as *x* approaches negative infinity.

Describe the end behavior of the graph of $f(x) = -0.5x^4 + 2.5x^2 + x - 1$
Solution:
Because the degree, 4, is even and the leading coefficient, -0.5, is negative
$f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$

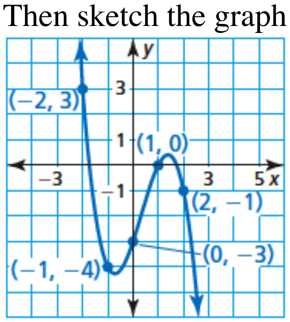
Evaluate the function $f(x) = 3x^5 - x^4 - 6x + 10$ at x = -2
Solution: $f(-2) = 3(-2)^5 - (-2)^4 - 6(-2) + 10$ = 3(-32) – (16) + 12 + 10 = -96 – 16 + 22 = $\boxed{-90}$.

To graph a polynomial function, we plot points to determine the shape of the middle portion. Then connect the points with a smooth continuous curve and use what we know about the end behavior to sketch the graph.

Graph: $f(x) = -x^3 + x^2 + 3x - 3$
The degree is odd and the leading coefficient is negative, so the end behavior is:
$f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$
Now, we make a table of some of the middle points and plot them.

x	−2	−1	0	1	2
$f(x)$	3	−4	−3	0	−1

Then sketch the graph.



Graph: $f(x) = x^4 - x^3 - 4x^2 + 4$

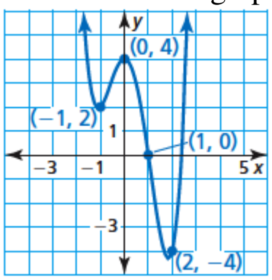
The degree is even and the leading coefficient is positive, so the end behavior is:

$f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$

Now, we make a table of some of the middle points and plot them.

x	−2	−1	0	1	2
f(x)	12	2	4	0	−4

Then sketch the graph.



When looking at a graph, we notice that each time the graph changes direction (increasing to decreasing etc.) we add another degree starting from one.

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Evaluate the function for the given value of x.

1. $h(x) = -3x^4 + 2x^3 - 12x - 6$; x = -2

2. $f(x) = 7x^4 - 10x^2 + 14x - 26$; x = -7

3. $g(x) = x^6 - 64x^4 + x^2 - 7x - 51$; $x = 8$

Describe the end behavior.

4. $h(x) = -5x^4 + 7x^3 - 6x^2 + 9x + 2$

5. $g(x) = 7x^7 + 12x^5 - 6x^3 - 2x - 18$

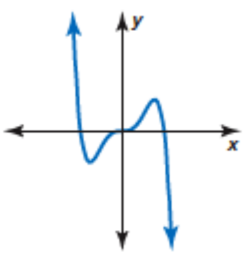
6. $f(x) = -2x^4 + 12x^8 + 17 + 15x^2$

7. $f(x) = 11 - 18x^2 - 5x^5 - 12x^4 - 2x$

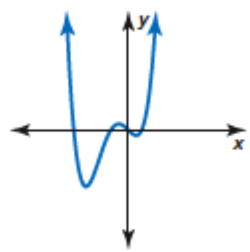
8. f(x)=13

Describe the degree and leading coefficient of the polynomial function using the graph.

9.



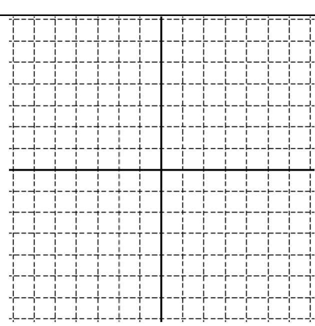
10.



Complete the table, then graph the polynomial function.

11. $g(x) = -x^3 + x^2 + 3x - 1$

x	-2	-1	0	1	2
g(x)					



12. $f(x) = x^3 - 3x^2 - 2$

x	-2	-1	0	1	2
f(x)