AP Calculus BC
135

The integral as an Accumulator

Suppose at time = 2, an object traveling in a straight line has position s(2) = 6. The object is moving at a rate of v(t) = 9sin(2t). Find the position of the object at time = 4. So far, in order to calculate this we would do the following:

Knowing that $\frac{ds(t)}{dt} = v(t)$, we can conclude that $s(t) = \int v(t)\,dt$

$s(t) = \int 9\sin(2t)\,dt = 9 \bullet \frac{1}{2}\int \sin(2t)(2\,dt) = -\frac{9}{2}\cos(2t) + C$

$s(2) = -\frac{9}{2}\cos(4) + C \approx 2.9414 + C = 6$

This allows us to find the value of C.
Therefore $C \approx 3.0586$

This gives a Function: $s(t) = -\frac{9}{2}\cos(2t) + 3.0586$

Finally, $s(4) \approx -\frac{9}{2}\cos(8) + 3.0586 \approx \boxed{3.71335}$.

Now Consider: $6 + \int_2^4 v(t)\,dt = 6 + \int_2^4 9\sin(2t)\,dt = 6 + \left[-\frac{9}{2}\cos(2t)\right]_2^4 = 6 - \frac{9}{2}\left[\left(\cos(8)\right) - \cos(4)\right] \approx \boxed{3.71335}$.

In general:
If $f(t)$ is differentiable on the interval [a, b] and $f'(t) = g(t)$, then the amount of change in $f(t)$ on the interval [a, b] is $\int_a^b g(t)\,dt$

Example:
Snow is accumulating on a driveway at the rate of $r(t) = 2t^{0.01} + e^{t/5}\ ft^3/\text{min}$.
Find the amount of showfall during the time t = 5 min to t = 11 min.
Solution:
$\int_5^{11}\left(2t^{0.01} + e^{t/5}\right)dt \approx \boxed{43.7828\,ft^3}$.

AP Calculus BC 1 Assignment 135 Friday, November 13, 2015 Hour Name

Calculator Active – Show all Set-Ups – Make all Answers Accurate to 3-Decimal Places

1. Find the area bounded by $f(x) = x + 1$, $g(x) = x^2 + 2$, $x = 1$, $x = 3$

2. Find the volume generated by revolution of the above region about the x-axis.

3. Find the volume generated by revolution of the above region about the y-axis.

4. A 25-foot ladder leaning against a vertical wall has its base being pulled away at rate of 2 feet per minute. At what rate is the top of the ladder moving when its base is 15 feet from the wall?

5. $f(x) = \frac{x^4}{4} - \frac{10\,x^3}{3} + 16x^2 - 32x + 12$. Find the x-coordinate of the relative maximum. Justify your answer.

6. A particle has velocity $v(t) = 4t^4 - 5t^3 + 6t^2 + 2t + 1$. How much is the displacement from the time $t = 3$ to the time $t = 6$.