

The Discriminant

In the Quadratic Formula, the expression inside the radical sign, $b^2 - 4ac$, is the Discriminant.

The Discriminant is important, because it can tell you the nature of the solutions for a Quadratic Equation. Often the Solutions are referred to as Roots.

The Discriminant must be in one of 3 states: it is less than 0, or equal to 0, or greater than 0. Each of these states gives a different nature of the solutions.

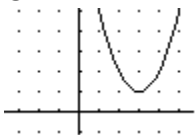
1. $b^2 - 4ac < 0 \rightarrow$ Two Imaginary Solutions (Roots)
2. $b^2 - 4ac = 0 \rightarrow$ One Real Solution (Root)
3. $b^2 - 4ac > 0 \rightarrow$ Two Real Solutions (Roots)

Examples – Find the Nature of the solutions, that is describe the number and type of roots:

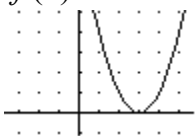
1. $x^2 - 6x + 10 = 0$
The Discriminant is $36 - 4(1)(10) = 36 - 40 = -4$. There are Two Imaginary Solutions.
2. $x^2 - 6x + 9 = 0$
The Discriminant is $36 - 4(1)(9) = 36 - 36 = 0$. There is Exactly One Real Solution.
3. $x^2 - 6x + 8 = 0$
The Discriminant is $36 - 4(1)(8) = 36 - 32 = 4$. There are Two Real Solutions.

If we look at Quadratic Functions instead of Quadratic Equations, the concept of Solutions becomes x-intercepts. Look at each of the examples above as Quadratic Functions.

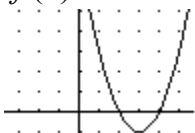
1. $f(x) = x^2 - 6x + 10$. The Discriminant indicates only Imaginary Solutions, there will be no x-intercepts. Check out the Graph Below.



2. $f(x) = x^2 - 6x + 9$. The Discriminant indicates Exactly one Real x-intercept.



3. $f(x) = x^2 - 6x + 8$. The Discriminant indicates Two Real x-intercepts.



Solve by Factoring:

1. $2x^2 - 11x + 12 = 0$

2. $5x^2 - 16x + 3 = 0$

3. $21x^2 + 4x = 1$

Solve by Completing the Square:

4. $2x^2 - 12x - 1 = 0$

5. $-3x^2 + 12x = 5$

6. $x^2 + 10x + 5 = 0$

Solve using the Quadratic Formula:

7. $2x^2 - 3x + 1 = 0$

8. $3x^2 + 4x - 2 = 0$

9. $2x^2 + 9x = 5$

Find the Number and Kind of Solutions:

10. $3x^2 - 4x + 2 = 10$

11. $x^2 + 6x + 10 = 0$

12. $4x^2 - 12x + 9 = 0$

Find the number of x-intercepts:

13. $f(x) = 3x^2 - 18x + 32$

14. $f(x) = 3x^2 - 6x - 2$

15. $f(x) = -2x^2 + 4x - 2$

Make a Quadratic Equation using the Quadratic Formula:

16. $x = \frac{-8 \pm \sqrt{-176}}{-10}$

17. $x = \frac{15 \pm \sqrt{-215}}{22}$

18. $x = \frac{-4 \pm \sqrt{-124}}{-14}$