Exponential and Logarithmic Models

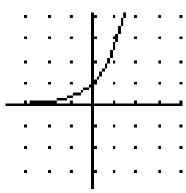
1. Exponential Growth Model: y = ae^{bx} — Asymptotic to –x-axis, y-int = a

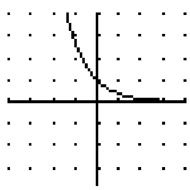
2. Exponential Decay Model: y = ae^{-bx} — Asymptotic to +x-axis, y-int = a

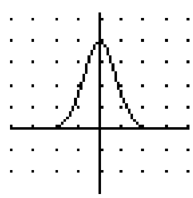
3. Gaussian Model: y = $ae^{-\frac{(x-b)^2}{c}}$ — Asymptotic to ±x-axis, Contains Point (b, a)

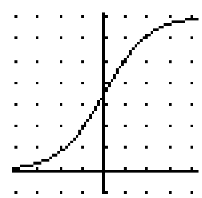
4. Logistic Growth Model: y = $\dfrac{a}{1+be^{-rx}}$ — Asymptotic to –x-axis & y = a, y-int = a

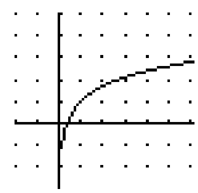
5. Logarithmic Models: y = $a+b\log_n x$ — Asymptotic to –y-axis, Contains Point (1, a)


Exp Growth


Exp Decay


Gaussian


Logistic


Logarithmic

Given the exponential growth relation, y = ae^{bx} that contains (2, 100) & (4, 300)
Find the missing component of (5, ?)

Using **Exponential Regression** on the calculator, we can get 519.6152423

Again: Given the exponential growth relation, $y = ae^{bx}$ that contains (2, 100) & (4, 300)
Now: Find the missing component of (5, ?) **without exponential regression**.

Solution:

$300 = ae^{4b}$ and $100 = ae^{2b}$ 2 Equations and 2 unknowns

$300 = ae^{4b}$ and $300 = 3ae^{2b}$

$ae^{4b} = 3ae^{2b}$

$e^{4b} = 3e^{2b}$

$\ln(e^{4b}) = \ln(3e^{2b}) = \ln(3) + \ln(e^{2b})$

$4b = \ln 3 + 2b$

$2b = \ln 3$

$b = \frac{1}{2}\ln 3$ *

$100 = ae^{\ln 3}$

$100 = 3a$

$a = \frac{100}{3}$ **

$y = \frac{100}{3}e^{\left(\frac{x}{2}\ln 3\right)}$

$y = \frac{100}{3}\left(e^{\ln 3}\right)^{\frac{x}{2}}$

$y = \frac{100}{3} \bullet 3^{\left(\frac{x}{2}\right)}$

$y = 100 \bullet 3^{x/2-1}$

$y(5) = 100 \bullet 3^{3/2}$

$\boxed{y(5) = 519.6152423}$

Convert an exponential expression with any base to an exponential with base e.
Suppose: $w^x = e^{bx}$ Find b.
$\ln(w^x) = \ln(e^{bx})$
$x \ln w = bx \ln e$
$x \ln w = bx$
$\boxed{\ln w = b}$

Suppose on the calculator you get an exponential equation: $y = aw^x$
Convert: $y = aw^x$ to $y = ae^{bx}$
Since $b = \ln w$, we get: $y = ae^{(\ln w)x}$

Example: Given a exponential model containing points (2, 6) and (6, 15).
Using the calculator, find the exponential model equation using e as the base.
The calculator gives $y = 3.7947\ (1.2574)^x$
To make the conversion, let $b = \ln(1.2574) = 0.229$
$\boxed{y = 3.7947\ e^{0.229x}}$

Also, find y when x = 10
Using the Exponential Regression from the calculator y = 37.5

Given: Half Life of a radio-active substance is 23 days. Count today as day 0 when there are Q grams of the substance. Write the equation that gives the amount of substance in t days:

a. Using $\frac{1}{2}$ as a base
b. Using e as a base

Solutions:

a. $A = Q\left(\frac{1}{2}\right)^{t/23}$, using a base of ½
b. $A = Qe^{\ln(1/2)\,t/23} \approx Qe^{-0.030t}$, using a base of e

A radio-active substance has a mass of 20 grams. It has a half life of 43 days. How long will it take for the amount of substance to become 8 grams?

$$8 = 20\left(\frac{1}{2}\right)^{t/43} \rightarrow \frac{2}{5} = \left(\frac{1}{2}\right)^{t/43} \rightarrow \ln\left(\frac{2}{5}\right) = \frac{t}{43}\ln\left(\frac{1}{2}\right) \rightarrow t = \frac{43\ln\left(\frac{2}{5}\right)}{\ln\left(\frac{1}{2}\right)} = \boxed{56.8439 \text{ Days}}$$

Assignment 131
Page 232, #'s 1-6, 23, 24, 30, 32, 34, 40