AP Calculus BC
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The Definite Integral and **The Fundamental Theorem of Calculus Part 1**

Whenever we have the Antiderivative Symbol with limits imposed on it, This symbol will be called the **Definite Integral**.
The Antiderivative Symbol without limits imposed on it may be called the **Indefinite Integral**.

The **Fundamental Theorem of Calculus (FTC)**
Given a Differentible Function F(x) on the interval (a, b) and F '(x) = f(x), where f(x) is continuous on [a, b].

Then: $\int_a^b f(x)\,dx = F(b) - F(a)$

Example: $\int_1^3 \left(3x^2 + 2x - 4\right)dx = (x^3 + x^2 - 4x)\Big|_1^3 = (27 + 9 - 12) - (1 + 1 - 4) = 24 + 2 = \boxed{26}$

Using a Riemann Sum we could find the area above the x-axis bounded by $f(x) = x^2 + 2x - 1$ on [2, 3].

$\Delta x = \frac{3-2}{n} = \frac{1}{n}$ = Width of each Rectangle

$f\left(2 + \frac{1}{n}i\right) = 4 + \frac{4}{n}i + \frac{1}{n^2}i^2 + 4 + \frac{2}{n}i - 1 = 7 + \frac{6}{n}i + \frac{1}{n^2}i^2$ = Height of Each Rectangle

Area = $\lim_{n \to \infty} \sum_{i=1}^{n} \frac{1}{n}\left[7 + \frac{6}{n}i + \frac{1}{n^2}i^2\right]$

$= \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \left[7 + \frac{6}{n}i + \frac{1}{n^2}i^2\right]$

$= \lim_{n \to \infty} \frac{1}{n}\left[7n + \frac{6}{n} \bullet \frac{n(n+1)}{2} + \frac{1}{n^2} \bullet \frac{n(n+1)(2n+1)}{6}\right]$

$= \lim_{n \to \infty} \left[7 + \frac{6n(n+1)}{2n^2} + \frac{n(n+1)(2n+1)}{6n^3}\right]$

$= 7 + 3 + \frac{1}{3} = 10 + \frac{1}{3} = \boxed{\frac{31}{3}}$

$\int_2^3 \left(x^2 + 2x - 1\right)dx = \left[\frac{1}{3}x^3 + x^2 - x\right]_2^3 = \left(\frac{27}{3} + 9 - 3\right) - \left(\frac{8}{3} + 4 - 2\right) = \frac{19}{3} + 9 - 3 - 4 + 2 = \frac{19}{3} + \frac{12}{3} = \boxed{\frac{31}{3}}$

We can now use the FTC 1 to find area.
Find the area bdd by y = sin x, y = 0, x = 0, and x = π.

Solution: $\int_0^\pi \sin x\,dx = -(\cos x)\Big|_0^\pi = -[\cos \pi - \cos 0] = -[-1 - 1] = \boxed{2}$

Interestingly, the area beneath the sine curve through half of its period is 2!!

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