$x^2 = 9$ has a real solution. The solution is $x = \pm 3$.

$x^2 = -1$ does not have a real solution, because if x is positive, x^2 is also positive, and if x is negative, x^2 is still positive. Therefore, no real number can be found to satisfy the equation.

Ancient mathematicians called the solution to the above equation "imaginary". This is unfortunate because some people take the word to suggest that the solution does not really exist. This is not true. Our current electronics rely on computations that use 'imaginary' numbers.

We define a solution to the above equation. $x = i$. As a consequence, $i = \sqrt{-1}$. When we look at consecutive powers of i, there is a pattern.

$i = i$

$i^2 = -1$

$i^3 = i^2 \bullet i = -i$

$i^4 = i^2 \bullet i^2 = 1$

$i^5 = i^4 \bullet i = i$

$i^6 = i^4 \bullet i^2 = -1$

$i^7 = i^4 \bullet i^3 = i^3 = -i$

$i^8 = i^4 \bullet i^4 = 1$

$i^9 = i^8 \bullet i = i$

$i^{10} = i^8 \bullet i^2 = -1$

$i^{11} = i^8 \bullet i^3 = -i$

$i^{12} = i^8 \bullet i^4 = 1$

Notice the pattern: $i, -1, -i, 1,\ i, -1, -i, 1,\ i, -1, -i, 1, \cdots$

Every power that is a multiple of 4 gives 1.

$i^{44} = 1$ because 44 is a multiple of 4.

Find i^{119}

Solution:

The last multiple leading up to 119 is 116. So $i^{119} = i^{116} \bullet i^3 = \boxed{-i}$

Simplify the Following by factoring to expose perfect squares.

1. $\sqrt{-36} = \sqrt{36} \bullet \sqrt{-1} = 6i$
2. $\sqrt{-4} = \sqrt{4} \bullet \sqrt{-1} = 2i$
3. $\sqrt{-18} = \sqrt{9}\sqrt{2}\sqrt{-1} = 3\sqrt{2}i$
4. $2\sqrt{27} = 2\sqrt{9}\sqrt{3}\sqrt{-1} = 2 \bullet 3\sqrt{3}\sqrt{-1} = 6\sqrt{3}\,i$
5. $-2\sqrt{-100} = -2\sqrt{100}\sqrt{-1} = -2 \bullet 10i = -20i$

Our number system is the set of complex numbers. The standard form for complex numbers is $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$. If $a = 0$, then the number is pure imaginary. If $b = 0$, then the number is real.

Find the values of *x* and *y* that satisfy the following equations.

1. $4x + 2i = 8 + yi$

   $4x = 8$ & $2i = yi$

   $\boxed{x = 2 \text{ \& } y = 2}$

2. $-3x - 5i = 12 + (y - 3)i$

   $-3x = 12$ & $-5 = y - 3$

   $\boxed{x = -4 \text{ \& } y = -2}$

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