

So far we've seen Horizontal Asymptotes when the Numerator has a smaller or equal degree compared to the degree of the denominator.

We will now look at SLANT ASYMPTOTES

Slant asymptotes occur when the numerator has a degree that is one higher than the denominator.

Example: $f(x) = \frac{4x^3 + 8x^2 + 2x - 5}{2x^2 + 5x + 1}$ Use Long Division

$$\begin{array}{r}
 2x - 1 \\
 \hline
 2x^2 + 5x + 1 \) \ 4x^3 + 8x^2 + 2x - 5 \\
 \underline{4x^3 + 10x^2 + 2x} \\
 -2x^2 \\
 \underline{-2x^2 - 5x - 1} \\
 5x - 4
 \end{array}$$

We will have the quotient: $2x - 1 + \frac{5x - 4}{2x^2 + 5x + 1}$

As x approaches $\pm \infty$, the remainder fraction gets closer and closer to zero and ultimately disappears.

Therefore the **Slant Asymptote** is: $\boxed{y = 2x - 1}$

At this point in our study, when faced with a Rational Function, we will calculate the following:

1. x-intercept
2. y-intercept
3. Horizontal or Slant Asymptote
4. Vertical Asymptote
5. Holes
6. Draw Graph

Example: $f(x) = \frac{2x^2 - 5x + 5}{x - 2}$ The numerator cannot be factored

$$\begin{array}{r} 2 \overline{) 2 \quad -5 \quad 5} \\ \underline{4 \quad -2} \\ 2 \quad -1 \quad 3 \end{array}$$

$$(2x^2 - 5x + 5) \div (x - 2) = 2x - 1 + \frac{3}{x - 2}$$

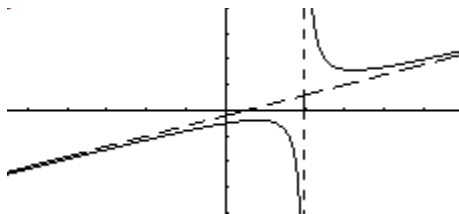
1. x-int:

2. y-int:

3. Slant A:

4. VA:

5. Hole:



For Each Exercise, On a Separate Sheet of Paper Find:

- a. x-int
- b. y-int
- c. Slant Asymptotes
- d. Vertical Asymptotes
- e. Holes

Box each item. If the item does not exist write None.

- f. Then Draw the Graph.

1. $f(x) = \frac{2x^2 + 1}{x}$

2. $g(x) = \frac{1 - x^2}{x}$

3. $h(x) = \frac{x^2}{x - 1}$

4. $g(x) = \frac{x^3}{2x^2 - 8}$

5. $f(x) = \frac{x^2 - 1}{x^2 + 4}$

6. $f(x) = \frac{x^3 + 2x^2 + 4}{2x^2 + 1}$