

## Parametric Equations

If  $x = f(t)$  and  $y = g(t)$ , then we have a pair of Parametric Equations. In this case  $t$  is called the Parameter.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}, \text{ where } \frac{dx}{dt} \neq 0$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \left( \frac{dx}{dt} \right) = \frac{\frac{d}{dt} \left( \frac{dy}{dx} \right)}{\frac{dx}{dt}}$$

Given:  $x = \sin t$ ,  $y = \cos t$

$$\frac{dy}{dx} = \frac{-\sin t}{\cos t} = -\tan t$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt} \left( \frac{dy}{dx} \right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt} (-\tan t)}{\frac{dx}{dt}} = \frac{-\sec^2 t}{\cos t} = -\sec^3 t$$

1.  $x = t^2$ ,  $y = 5 - 4t$  Find  $\frac{dy}{dx}$ .

2.  $x = \sqrt[3]{t}$ ,  $y = 4 - t$  Find  $\frac{dy}{dx}$ .

3.  $x = \sin^2\theta$ ,  $y = \cos^2\theta$  Find  $\frac{dy}{dx}$ .

4.  $x = 2e^\theta$ ,  $y = e^{\theta/2}$  Find  $\frac{dy}{dx}$ .

5.  $x = 2t$ ,  $y = 3t - 1$  At  $t = 3$ , Find  $\frac{dy}{dx}$ .

6.  $x = \sqrt{t}$ ,  $y = 3t - 1$  At  $t = 3$ , Find  $\frac{d^2y}{dx^2}$ .

7.  $x = t + 1$ ,  $y = t^2 + 3t$  At  $t = -1$ , Find  $\frac{d^2y}{dx^2}$ .

8.  $x = t^2 + 3t + 2$ ,  $y = 2t$  At  $t = 0$ , Find  $\frac{d^2y}{dx^2}$ .

9.  $x = 2 \cos \theta$ ,  $y = 2 \sin \theta$  At  $t = \frac{\pi}{4}$ , Find  $\frac{d^2y}{dx^2}$ .

10.  $x = 2 + \sec \theta$ ,  $y = 1 + 2 \tan \theta$  At  $\theta = \frac{\pi}{6}$ , Find  $\frac{d^2y}{dx^2}$ .