

1. A monthly teen magazine has 48,000 subscribers when it charges \$20 per annual subscription. For each \$1 increase in price, the magazine loses about 2000 subscribers. How much should the magazine charge to maximize annual revenue? What is the maximum annual revenue?

Solution:

- Define the variables: Let x represent the price increase.
Let $R(x)$ represent the annual revenue.
- Write a verbal model. Then write and simplify a quadratic function.

Annual Revenue (Dollars)	Number of Subscribers (People)	Subscription Price (Dollard/Person)
$R(x)$	$= (48,000 - 2000x)$	$\cdot (20 + x)$
$R(x)$	$= (-2000x + 48,000)(x + 20)$	
$R(x)$	$= -2000(x - 24)(x + 20)$	

- Identify the zeros and find their average. Then find how much each subscription should cost to maximize annual revenue

The zeros of the revenue function are 24 and -20. The average of the zeros is $\frac{24 + (-20)}{2} = 2$.

To maximize revenue, each subscription should cost $\$20 + \$2 = \$22$.

- Find the maximum annue revenue.

$$R(2) = -2000(2 - 24)(x + 20) = \$968,000$$

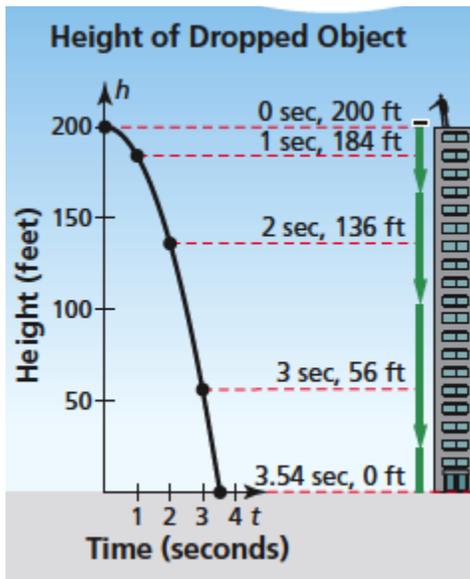
So, the magazine should charge **\$22 per subscription** to maximize annual revenue. The maximum revenue is **\$968,000**.

2. A monthly skateboard magazine has 50,000 subscribers when it charges \$12 per annual subscription. For each \$1 increase in price, the magazine loses about 2500 subscribers. How much should the magazine charge to maximize annual revenue? What is the maximum annual revenue?

Solution

Annual Revenue (Dollars)	Number of Subscribers (People)	Subscription Price (Dollard/Person)
$R(x)$	$= (50,000 - 2500x)$	$\cdot (12 + x)$
$R(x)$	$= (-2500x + 50,000)(x + 12)$	
$R(x)$	$= -25000(x - 20)(x + 12)$	

The Maximum for $R(x)$, midway between 20 & -12, is 4. $R(4) = -25000(-16)(16) = \$640,000$.
The Charge per magazine should be $12 + 4 = \mathbf{\$16.00}$... $R(4) = -25000(-16)(16) = \mathbf{\$640,000}$.



When an object is dropped, its height h (in feet) above the ground after t seconds can be modeled by the function $h = -16t^2 + h_0$, where h_0 is the initial height (in feet) of the object. The graph of $h = -16t^2 + 200$, representing the height of an object dropped from an initial height of 200 feet, is shown at the left.

The model $h = -16t^2 + h_0$ assumes that the force of air resistance on the object is negligible. Also this model applies only to objects dropped on Earth. For planets with stringer or weaker gravitational forces, different models are used.

For a science competition, students must design a container that prevents an egg from breaking when dropped from a height of 50 feet.

- Write a function that gives the height h (in feet) of the container after t seconds. How long does the container take to hit the ground?
- Find and interpret $h(1) - h(1.5)$.

Solution:

- $h_0 = 50$, so we have $h = -16t^2 + 50$. To hit the ground, we must find the zeros of the function.

$$0 = -16t^2 + 50 \rightarrow 16t^2 = 50 \rightarrow t^2 = \frac{50}{16} = \frac{25}{8} \rightarrow t = \pm\sqrt{\frac{25}{8}} = \boxed{\pm 1.8 \text{ seconds}}$$

- $h(1) = -16(1)^2 + 50 = 34$ & $h(1.5) = -16(1.5)^2 + 50 = 14$ $h(1) - h(1.5) = 20$ This is the distance the container fell from time = 1 to time = 1.5 seconds

57. A restaurant sells 330 sandwiches each day for \$6.00 Each. For each \$0.25 decrease in price, the restaurant sells about 15 more sandwiches. How much should the restaurant charge to maximize daily revenue? What is the maximum daily revenue?
58. An athletic store sells about 200 pairs of basketball shoes per month when it charges \$120 per pair. For each \$2 increase in price, the store sells two fewer pairs of shoes. How much should the store charge to maximize monthly revenue? What is the maximum monthly revenue?
59. Niagara Falls is made up of three waterfalls. The height of the Canadian Horseshoe Falls is about 188 feet above the lower Niagara River. A log falls from the top of Horseshoe Falls.
- Write a function that gives the height h (in feet) of the log after t seconds. How long does the log take to reach the river?
 - Find and interpret $h(2) - h(3)$.

60. According to legend, in 1589, the Italian scientist Galileo Galilei dropped rocks of different weights from the top of the Leaning Tower of Pisa to prove his conjecture that the rocks would hit the ground at the same time. The height h (in feet) of a rock after t seconds can be modeled by $h(t) = 196 - 16t^2$.
- Find and interpret the zeros of the function. Then use the zeros to sketch the graph.
 - What do the domain and range of the function represent in this situation?
61. You make a rectangular quilt that is 5 feet by 4 feet. You use the remaining 10 square feet of fabric to add a border of uniform width to the quilt. What is the width of the border?
62. You drop a seashell into the ocean from a height of 40 feet. Write an equation that models the height h (in feet) of the seashell above the water after t seconds. How long is the seashell in the air?

57. A restaurant sells 330 sandwiches each day for \$6.00 Each. For each \$0.25 decrease in price, the restaurant sells about 15 more sandwiches. How much should the restaurant charge to maximize daily revenue? What is the maximum daily revenue?

$$x = \# \text{ of Quarters decrease. } R(x) = (330 + 15x)(6 - 0.25x) = 15(x + 22)(0.25)(24 - x)$$

$$= -3.75(x + 22)(x - 24) \text{ Max at } x = 1 \text{ quarter. } \boxed{\text{Charge } \$5.75 \text{ Each } R(1) = \$1983.75 \text{ Max Revenue}}$$

58. An athletic store sells about 200 pairs of basketball shoes per month when it charges \$120 per pair. For each \$2 increase in price, the store sells two fewer pairs of shoes. How much should the store charge to maximize monthly revenue? What is the maximum monthly revenue?

$$x = \# \text{ of } \$2 \text{ increase. } R(x) = (200 - 2x)(120 + 2x) = 4(100 - x)(60 + x) = -4(x - 100)(x + 60)$$

$$\text{Max at } x = 20 \text{ (}\$2.00 \text{ increases). } \boxed{\text{Charge } \$160 \text{ Each Pair } R(20) = \$25,600.00 \text{ Max Revenue}}$$

59. Niagara Falls is made up of three waterfalls. The height of the Canadian Horseshoe Falls is about 188 feet above the lower Niagara River. A log falls from the top of Horseshoe Falls.
- Write a function that gives the height h (in feet) of the log after t seconds. How long does the log take to reach the river?
 - Find and interpret $h(2) - h(3)$.

$$h(t) = 188 - 16t^2 \quad 0 = 188 - 16t^2 \quad t^2 = \frac{47}{4} \quad t = 3.42783 \text{ Seconds}$$

$$h(2) = 188 - 64 = 124 \quad h(3) = 188 - 144 = 44 \quad \boxed{h(2) - h(3) = 80 \text{ ft fallen between seconds 2 \& 3.}}$$