

1. If  $c$  is a zero of the denominator, but not the numerator, then  $x = c$  is a **vertical asymptote**. If the factor  $(x-c)$  occurs an even number of times, we have an even asymptote. If it occurs an odd number of times, we have a odd asymptote. With an even asymptote, the graph approaches it the same way on each side, either high and high or low and low. With an odd asymptote, the graph approaches in opposite directions.
2. If  $a$  is a zero of the numerator, but not the denominator, then  $a$  is an **x-intercept**.
3. If  $(x - h)$  can be cancelled out of the numerator and denominator, leaving a function where  $h$  can be used, then  $h$  as the **x-coordinate of a hole**.
4. Replace all  $x$ -values with zero and evaluate the function will give the **y-intercept**.

5.  $\lim_{x \rightarrow \pm\infty} f(x)$  will give the **Non-Vertical Asymptote(s)**.

$$6. \quad f(x) = \frac{2x^2 + 2x - 12}{x^2 + x - 30} = \frac{2(x^2 + x - 6)}{x^2 + x - 30} = \frac{2(x+3)(x-2)}{(x+6)(x-5)}$$

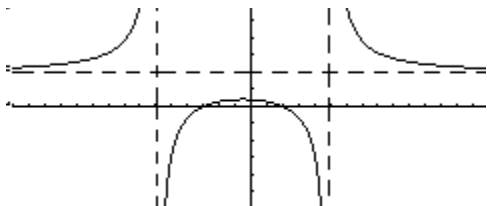
x-int: -3, 2

y-int:  $\frac{2}{5}$

Horiz Asy:  $y = 2$

Vert Asy:  $x = -6$  &  $x = 5$  Both are Odd Asymptotes

Hole: None



$$7. \quad f(x) = \frac{x^2 - 9}{x^2 - 2x - 3} = \frac{(x+3)(x-3)}{(x-3)(x+1)} = \frac{x+3}{x+1}, \quad x \neq 3$$

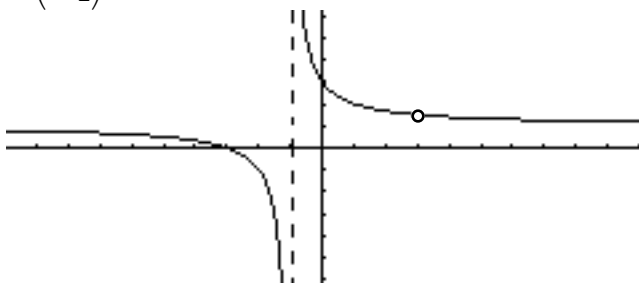
x-int: -3

y-int: 3

HA:  $y = 1$

VA:  $x = -1$  Odd Asymptote

Holes:  $(3, \frac{3}{2})$



For Each Exercise, On **Separate Sheets** of Paper Find:

- a. x-int
- b. y-int
- c. Horizontal Asymptotes
- d. Vertical Asymptotes
- e. Holes

Box each item. If the item does not exist write None.

- f. Then Draw the Graph.

1. 
$$f(x) = \frac{(x+2)(x-3)(x-1)}{(x-1)(x-5)^2}$$

2. 
$$f(x) = \frac{x}{x^2 - 4}$$

3. 
$$f(x) = \frac{x^2 - 4}{x^2 - 4x}$$

4. 
$$f(x) = \frac{2x}{x^2 + x - 2}$$

5. 
$$f(x) = \frac{3x^2 - 8x + 4}{2x^2 - 3x - 2}$$

6. 
$$f(x) = \frac{x^2 - 1}{x^2 + 4}$$