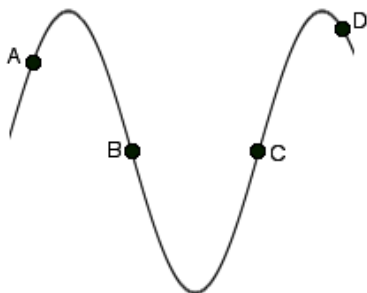


Recall that Derivative  $\leftrightarrow$  Rate of Change  $\leftrightarrow$  Slope

The Second Derivative of a function,  $y(x)$ , is written as  $\frac{d^2 y}{dx^2}$ .

This second derivative gives the:

Rate of Change of the Slope  $\leftrightarrow$  Rate of Change of the Derivative  $\leftrightarrow$  Slope of the Derivative.



Along the Graph from A to B, The Slope becomes less positive, then negative. The Rate of Change of the Slope is Negative. On that interval, the change of the derivative is negative. That is, the Second Derivative of the Function is Negative. Also, note that on this interval, the curve is Concave Down.

Also Note that along the graph from B to C The Derivative is changing in a positive direction. The second derivative is positive. The curve is Concave Up.

At B and C, the Concavity Changes from Down to Up and From Up to Down respectively. Where the concavity changes is referred to as a Point of Inflection.

In Calculus Terms, a Point of Inflection can only occur on a Twice Differentiable Function where the Second Derivative Changes Sign.

Note: The Second Derivative = Zero is necessary Condition for a Point of Inflection, but not a Sufficient Condition.

It is possible to find a point where the 2<sup>nd</sup> Derivative is zero, without a Point of Inflection.

The fact that the 2<sup>nd</sup> Derivative can give concavity allows us to use the Second Derivative Test for Relative Extrema.

### **The Second Derivative Test**

A Relative Maximum Exists at  $x = c$  on a Twice Differentiable Function,  $f(x)$ , if both are true:

1.  $f'(c) = 0$
2.  $f''(c) < 0$

A Relative Minimum Exists at  $x = c$  on a Twice Differentiable Function,  $f(x)$ , if both are true:

1.  $f'(c) = 0$
2.  $f''(c) > 0$