

When data have **equally-spaced** inputs, you can analyze **patterns** in the **differences** of the outputs to determine what type of function can be used to model the data. Linear data have constant first differences. **Quadratic** data have **constant second differences**. The first and second differences of $f(x) = x^2$ are shown below.

Equally-spaced x-values

x	-3	-2	-1	0	1	2	3
$f(x)$	9	4	1	0	1	4	9

first differences: -5 -3 -1 1 3 5

second differences: 2 2 2 2 2

Because the 2nd Differences are all the same, The Data can be modeled by a Quadratic Function.

All Quadratic Functions can take on the form:

$$f(x) = ax^2 + bx + c$$

To specify a function, we need to know a , b , and c . So we essentially have 3 unknowns and therefore we need 3 equations in a system to solve for them. Let's pick any 3 pairs of numbers from the above table.

$(0, 0)$, $(1, 1)$, $(2, 4)$. These can replace x and y in our equations.

$$\begin{array}{l} \text{Use } (0,0) \text{ Let } x = 0 \text{ \& } y = 0 \\ \text{Use } (1,1) \text{ Let } x = 1 \text{ \& } y = 1 \\ \text{Use } (2,4) \text{ Let } x = 2 \text{ \& } y = 4 \end{array} \rightarrow \begin{cases} a(0) + b(0) + c = 0 \\ a(1) + b(1) + c = 1 \\ a(2) + b(2) + c = 4 \end{cases} \rightarrow \begin{cases} c = 0 \\ a + b + c = 1 \\ 4a + 2b + c = 4 \end{cases} \text{ In this case we already know } c = 0.$$

$$\begin{cases} a + b = 1 \\ 4a + 2b = 4 \end{cases} \text{ Multiply the top by } -2. \quad \begin{cases} -2a - 2b = -2 \\ 4a + 2b = 4 \end{cases} \text{ Add the Equations and cancel the } b \text{ terms.}$$

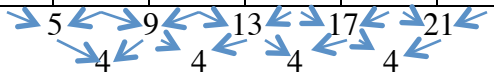
$2a = 2$ So $a = 1$ We can now find b by substitution of 0 for c , and 1 for a .

$$a + b = 1 \quad 1 + b = 1 \quad b = 0.$$

The Function: $f(x) = x^2$.

Find the Function that includes the following data.

x	2	3	4	5	6	7
f(x)	1	6	15	28	45	66



First Differences
Second Differences

Since the 2nd Differences are all 4, we have a quadratic Function. Pick any 3 ordered pairs. (2, 1), (3, 6), (4, 15) These will replace x and y in $ax^2 + bx + c = y$ to make 3 equations.

$$\begin{cases} 4a + 2b + c = 1 \\ 9a + 3b + c = 6 \\ 16a + 4b + c = 15 \end{cases} \quad \text{Multiply the 2nd and 3rd equations by -1 to cancel the c's.}$$

$$\begin{cases} 4a + 2b + c = 1 \\ -9a - 3b - c = -6 \\ -16a - 4b - c = -15 \end{cases} \quad \text{Add Equations 1 \& 2. Also Add Equations 1 and 3.}$$

$$\begin{cases} -5a - b = -5 \\ -12a - 2b = -14 \end{cases} \quad \text{Multiply Equation 1 by -2.}$$

$$\begin{cases} 10a + 2b = 10 \\ -12a - 2b = -14 \end{cases} \quad \text{Add the Equations to cancel the b's.}$$

$$-2a = -4 \text{ so } a = 2.$$

$$\text{To solve for b, replace a with 2. } -5(2) - b = -5 \quad -b = 5 \quad b = -5$$

$$\text{Now solve for c: } 4(2) + 2(-5) + c = 1 \quad 8 - 10 + c = 1 \quad c = 1 + 2 \quad c = 3$$

$$\text{The Function: } \boxed{f(x) = 2x^2 - 5x + 3}.$$

1. Find the Quadratic Function for the Table:

x	2	3	4	5
f(x)	0	2	6	12

2. Find the Quadratic Function for the Table:

x	-1	0	1	2
f(x)	-4	-1	-2	-7

3. Find the Quadratic Function for the Table:

x	-1	0	1	2
f(x)	6	2	4	12

4. Find the Quadratic Function for the Table:

x	0	1	2	3
f(x)	3	2	5	12