Algebra 2

121

When data have **equally-spaced** inputs, you can analyze **patterns** in the **differences** of the outputs to determine what type of function can be used to model the data. Linear data have constant first differences. **Quadratic** data have **constant second differences**. The first and second differences of f(x) = x^2 are shown below.

Equally-spaced *x*-values

x	−3	−2	−1	0	1	2	3
f(x)	9	4	1	0	1	4	9

first differences: −5 −3 −1 1 3 5

second differences: 2 2 2 2 2

Because the 2nd Differences are all the same, The Data can be modeled by a Quadratic Function.

All Quadratic Functions can take on the form:

$f(x) = ax^2 + bx + c$

To specify a function, we need to know *a, b,* and *c*. So we essentially have 3 unknowns and therefore we need 3 equations in a system to solve for them. Let's pick any 3 pairs of numbers from the above table.
(0, 0), (1, 1), (2, 4). These can replace x and y in our equations.

$$\begin{array}{l} Use\ (0,0)\ Let\ x = 0\ \&\ y = 0 \\ Use\ (1,1)\ Let\ x = 1\ \&\ y = 1 \\ Use\ (2,4)\ Let\ x = 2\ \&\ y = 4 \end{array} \rightarrow \begin{cases} a(0) + b(0) + c = 0 \\ a(1) + b(1) + c = 1 \\ a(4) + b(2) + c = 4 \end{cases} \rightarrow \begin{cases} c = 0 \\ a + b + c = 1 \\ 4a + 2b + c = 4 \end{cases}$$

In this case we already know c = 0.

$\begin{cases} a + b = 1 \\ 4a + 2b = 4 \end{cases}$ Multiply the top by -2. $\begin{cases} -2a - 2b = -2 \\ 4a + 2b = 4 \end{cases}$ Add the Equations and cancel the *b* terms.

$2a = 2$ So $a = 1$ We can now find *b* by substitution of 0 for *c*, and 1 for *a*.

$a + b = 1 \quad 1 + b = 1 \quad b = 0.$

The Function: $\boxed{f(x) = x^2}$.

Find the Function that includes the following data.

x	2	3	4	5	6	7
f(x)	1	6	15	28	45	66

5 9 13 17 21 First Differences

4 4 4 4 Second Differences

Since the 2nd Differences are all 4, we have a quadratic Function. Pick any 3 ordered pairs.
(2, 1), (3, 6), (4, 15) These will replace x and y in $ax^2 + bx + c = y$ to make 3 equations.

$$\begin{cases} 4a + 2b + c = 1 \\ 9a + 3b + c = 6 \\ 16a + 4b + c = 15 \end{cases}$$

Multiply the 2nd and 3rd equations by -1 to cancel the c's.

$$\begin{cases} 4a + 2b + c = 1 \\ -9a - 3b - c = -6 \\ -16a - 4b - c = -15 \end{cases}$$

Add Equations 1 & 2. Also Add Equations 1 and 3.

$$\begin{cases} -5a - b = -5 \\ -12a - 2b = -14 \end{cases}$$

Multiply Equation 1 by -2.

$$\begin{cases} 10a + 2b = 10 \\ -12a - 2b = -14 \end{cases}$$

Add the Equations to cancel the b's.

-2a = -4 so a = 2.
To solve for b, replace a with 2. -5(2) – b = -5 -b = 5 b = -5
Now solve for c: 4(2) + 2(-5) + c = 1 8 – 10 + c = 1 c = 1 + 2 c = 3
The Function: $\boxed{f(x) = 2x^2 - 5x + 3}$.

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1. Find the Quadratic Function for the Table:

x	2	3	4	5
f(x)	0	2	6	12

2. Find the Quadratic Function for the Table:

x	-1	0	1	2
f(x)	-4	-1	-2	-7

3. Find the Quadratic Function for the Table:

x	-1	0	1	2
f(x)	6	2	4	12

4. Find the Quadratic Function for the Table:

x	0	1	2	3
f(x)	3	2	5	12