1. Fundamental Theorem of Algebra: If f(x) is a polynomial of degree n, $n > 0$, then f(x) has at least one complex zero.

2. Linear Factorization Theorem: If f(x) is a polynomial of degree n, $n > 0$, then f(x) has precisely n linear factors with complex constants.

3. Complex Conjugate Theorem: If f(x) is a real coefficient polynomial and $a + bi$ is a zero, then $a - bi$ is also a zero.

4. Every real coefficient polynomial can be written as the product of linear and quadratic factors with real coefficients, where the quadratic factors have no real zeros.

5. Find all zeros of $f(x) = x^2 - 4x + 1$, then write f(x) as a product of linear factors.

$$x^2 - 4x + 1 = 0$$

$$x = \frac{4 \pm \sqrt{16 - 4(1)(1)}}{2} = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = \boxed{2 \pm \sqrt{3} \text{ These are the zeros}}.$$

$$\boxed{f(x) = \left(x - 2 - \sqrt{3}\right)\left(x - 2 + \sqrt{3}\right) \text{ This is the product of linear factors}}$$

6. Write $f(x) = 3x^3 - 5x^2 + 48x - 80$ as a product of linear factors.

$\frac{5}{3}$	3	-5	48	-80
		5	0	80
	3	0	48	0

$$f(x) = \left(x - \frac{5}{3}\right)\left(3x^2 + 48\right)$$

$$f(x) = \left(x - \frac{5}{3}\right)(3)(x^2 + 16)$$

$$f(x) = \left(x - \frac{5}{3}\right)(3)(x + 4i)(x - 4i)$$

$$f(x) = \boxed{(3x - 5)(x + 4i)(x - 4i)}$$

7. Find a polynomial with real coefficients with the following zeros: 2 and $3 - 4i$

We also know that $3 + 4i$ is also a zero.

$f(x) = (x - 2)(x - 3 + 4i)(x - 3 - 4i)$

$f(x) = (x - 2)([x - 3] + 4i)([x - 3] - 4i)$

$f(x) = (x - 2)([x - 3]^2 - 16i^2)$

$f(x) = (x - 2)(x^2 - 6x + 9 - 16i^2)$

$f(x) = (x - 2)(x^2 - 6x + 25)$

$f(x) = x^3 - 6x^2 + 25x - 2x^2 + 12x - 50$

$f(x) = \boxed{x^3 - 8x^2 + 37x - 50}$

8. Find a polynomial with real coefficients with the following zeros: $0, 4, 1 + \sqrt{2}\, i$

$$\boxed{f(x) = x^4 - 6x^3 + 11x^2 - 12x}$$

9. $5 + 2i$ is a zero of $f(x) = x^3 - 7x^2 - x + 87$. Find all the zeros.

Since the polynomial has real coefficients, we know that $5 - 2i$ is also a zero.
We now know that $(x - 5 - 2i)(x - 5 + 2i)$ divides evenly into $x^3 - 7x^2 - x + 87$

$x^2 - 10x + 25 - 4i^2$ divides evenly into $x^3 - 7x^2 - x + 87$
$x^2 - 10x + 29$ divides evenly into $x^3 - 7x^2 - x + 87$

$$
\begin{array}{r l}
 & x + 3 \\
x^2 - 10x + 29 \,/ & \overline{x^3 - 7x^2 - x + 87} \\
 & \underline{x^3 - 10x^2 + 29x} \\
 & \quad 3x^2 - 30x + 87 \\
 & \quad \underline{3x^2 - 30x + 87} \\
 & \qquad 0
\end{array}
$$

This means that $x + 3$ is also a factor

Zeros are: $5 \pm 2i$ and -3

10. $1 - 3i$ is a zero of $f(x) = x^4 - 3x^3 + 6x^2 + 2x - 60$. Find all the zeros.

We know that $1 + 3i$ is also a zero

$[(x - 1) + 3i][(x - 1) - 3i]$ divides evenly into $x^4 - 3x^3 + 6x^2 + 2x - 60$

$x^2 - 2x + 1 - 9i^2$ divides evenly into $x^4 - 3x^3 + 6x^2 + 2x - 60$

$x^2 - 2x + 10$ divides evenly into $x^4 - 3x^3 + 6x^2 + 2x - 60$

$$
\begin{array}{r l}
 & x^2 - x - 6 \\
x^2 - 2x + 10 \,/ & \overline{x^4 - 3x^3 + 6x^2 + 2x - 60} \\
 & \underline{x^4 - 2x^3 + 10x^2} \\
 & \quad -x^3 - 4x^2 + 2x - 60 \\
 & \quad \underline{-x^3 + 2x^2 - 10x} \\
 & \qquad -6x^2 + 12x - 60 \\
 & \qquad \underline{-6x^2 + 12x - 60} \\
 & \qquad\qquad 0
\end{array}
$$

$x^2 - x - 6 = (x - 3)(x + 2) = 0 \rightarrow$ Zeros are 3, and -2

Zeros are: $1 \pm 3i$, 3, and -2

Assignment 120
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