Complex Numbers

$x^2 + 1 = 0$

1. If x is positive, the statement is not true.
2. If x is negative, the statement is not true.
3. If x = 0, the statement is not true.
4. Therefore, this equation does not have a real solution.

The only way for this equation to have a solution is to allow $x^2 = -1$.

We now introduce a number, called "i", where $i^2 = -1$.
As a result, we will calculate $i = \sqrt{-1}$

Let's look for a pattern:

1. $i^1 = \boxed{i}$
2. $i^2 = \boxed{-1}$
3. $i^3 = i^2 \bullet i = (-1)i = \boxed{-i}$
4. $i^4 = i^2 \bullet i^2 = (-1)(-1) = \boxed{1}$
5. $i^5 = i^4 \bullet i = (1)i = \boxed{i}$
6. $i^6 = i^2 \bullet i^4 = (-1)(1) = \boxed{-1}$
7. $i^7 = i^3 \bullet i^4 = (-i)(1) = \boxed{-i}$
8. $i^8 = i^4 \bullet i^4 = (1)(1) = \boxed{1}$
9. $i^9 = i^8 \bullet i = (1)(i) = \boxed{i}$
10. $i^{10} = i^8 \bullet i^2 = (1)(-1) = \boxed{-1}$
11. $i^{11} = i^8 \bullet i^3 = (1)(-i) = \boxed{-i}$
12. $i^{12} = i^8 \bullet i^4 = (1)(1) = \boxed{1}$
13. $i^{13} = i^{12} \bullet i = (1)(i) = \boxed{i}$
14. $i^{14} = i^{12} \bullet i^2 = (1)(-1) = \boxed{-1}$
15. $i^{15} = i^{12} \bullet i^3 = (1)(-i) = \boxed{-i}$
16. $i^{16} = i^{12} \bullet i^4 = (1)(1) = \boxed{1}$

Notice the pattern for powers of i. We have a cycle of 4.

Powers of i	1	2	3	4	5	6	7	8	9	10	11	12
Outcomes	i	-1	-i	1	i	-1	-i	1	i	-1	-i	1

Each exponent that is a multiple of 4 gives 1.

1. Find i^{47}

   $i^{47} = i^{44} \bullet i^3 = (1)(-i) = \boxed{-i}$ We picked 44 because 44 is the last multiple of 4 before 47

2. Find i^{102}

   $i^{102} = i^{100} \bullet i^2 = (1)(-1) = \boxed{-1}$

3. Find i^{69}

   $i^{69} = i^{68} \bullet i = (1)(i) = \boxed{i}$

A **Complex Number** is a number that **may** be written in the form:
a + bi, where a and b are real numbers.

This form is called **Standard Form**. When **b = 0**, then the Standard Form is just **a**.

Write the following in standard form:

1. $(2 - 3i) + (5 + 8i)$

   $\boxed{7 + 5i}$

2. $(3 + 4i)(2 - 3i)$

   $6 - 9i + 8i - 12i^2$

   $6 - i + 12$

   $\boxed{18 - i}$

3. $2i(3 + 5i)$

   $6i + 10i^2$

   $6i - 10$

   $\boxed{-10 + 6i}$

4. $(5 – 6i)(5 + 6i)$

   $25 + 30i – 30i – 36i^2$

   $25 – 36(-1)$

   $25 + 36$

   $\boxed{61}$

If a + bi is a complex number, then a – bi is its **Complex Conjugate.**

When a pair of complex conjugtes are multiplied, you get a Real Number. The **i** Terms Cancel Out.

$(a - bi)(a + bi) = a^2 + abi - abi - b^2i^2 = a^2 - b^2(-1) = a^2 + b^2$

Rewrite the Following in Standard Form.

5. Rewrite in Standard Form: $\frac{2 + 3i}{5 - 2i}$

   Multiply the top and bottom by the **complex conjugate** of the denominator.

   $$\frac{2 + 3i}{5 - 2i} \bullet \frac{5 + 2i}{5 + 2i}$$

   $$\frac{10 + 4i + 15i + 6i^2}{25 - 4i^2}$$

   $$\frac{10 + 19i - 6}{25 + 4}$$

   $$\frac{4 + 19i}{29}$$

   $$\boxed{\frac{4}{29} + \frac{19}{29}i}$$

6. $2 - \sqrt{-25}$

   $2 - \sqrt{25} \bullet \sqrt{-1}$

   $\boxed{2 - 5i}$

7. $(-1 + \sqrt{-8}) + (8 - \sqrt{-50})$

   $\left(-1 + 2\sqrt{2}\, i\right) + \left(8 - 5\sqrt{2}\, i\right)$

   $\boxed{7 - 3\sqrt{2}\, i}$

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