

Extrema

If f is a continuous function on the interval (p, q) and $[a, b]$ is contained in (p, q) , then f has a maximum value and a minimum value on $[a, b]$. These values are called **Absolute Extrema** or **Extrema**.

If c is on (d, e) and $f(c) > f(x)$ for all x in (d, e) , then $f(c)$ is a **Relative Maximum**.

If c is on (d, e) and $f(c) < f(x)$ for all x in (d, e) , then $f(c)$ is a **Relative Minimum**.

The Derivative of a continuous function at these **Relative Extrema** is either **Zero** or **The Derivative Does Not Exist**.

If $f'(c) = 0$, or If $f'(c)$ does not exist on a function that is continuous at c , then c is called a **Critical Number**.

Relative Extrema only occur at **Critical Numbers**.

To find Extrema of f on a closed interval $[a, b]$:

1. Find the critical values on (a, b) . These are values where $f' = 0$ or f' does not exist.
2. Evaluate each f at each critical value.
3. Evaluate f at each endpoint of $[a, b]$.
4. The least of these value is the minimum and the greatest of these values is the maximum.

Find the extrema of $f(x) = 3x^4 - 4x^3$ on $[-1, 2]$.

$$1. \quad f'(x) = 12x^3 - 12x^2 \quad 12x^3 - 12x^2 = 0 \quad 12x^2(x - 1) = 0 \quad x = 0 \text{ \& } x = 1$$

These are the **Critical Values**.

2. $f(0) = 0$ and $f(1) = -1$.
3. $f(-1) = 7$ and $f(2) = 16$
4. The minimum value is -1 and the maximum value is 16.

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