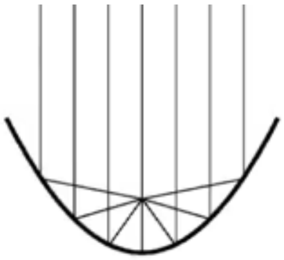
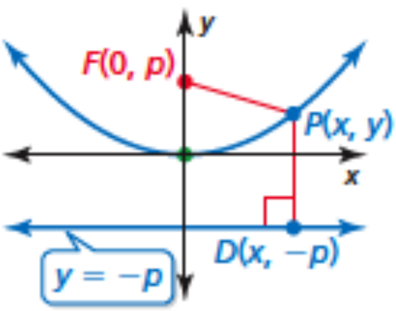
Parabolas have a point called the Focus. If you have light rays entering a parabolic surface parallel to the axis of symmetry, they will all converge at the Focus.



This is how a satellite dish works. Signals coming in will converge at the focus. So communications technicians place the signal pick-up at the focus point. A typical flashlight uses the same principle, but in reverse. The bulb is placed at the focus so that the light reflects off the parabolic mirror and sends the light straight ahead.

Along with the Focus, there is another element, a line, associated with the parabola called directrix.
As it turns out, every point on the parabola is equidistant from the directrix and the focus.



In the figure above, the focus F(0, p) is on the y-axis. The directrix has equation y = -p. The distance between the Focus to a point on the parabola is FP. The distance between the Directrix and that point on the parabola is PD. So PD = PF. The distance formula applied to each side gives the following.

$$\sqrt{(x-x)^2+(y-(-p))^2} = \sqrt{(x-0)^2+(y-p)^2}$$

$$(y+p)^2 = x^2+(y-p)^2$$

$$y^2+2py+p^2 = x^2+y^2-2py+p^2$$

$$4py = x^2$$

$y = \dfrac{1}{4p}x^2$ This equation is for a parabola that has a vertex (0, 0) and opens up or down.

If the parabola opens right or left and had the vertex at (0, 0), the equation is $x = \dfrac{1}{4p}y^2$.

The **<u>Directed Distance</u>** from the Vertex to the Focus is p. The **<u>Directed Distance</u>** from the Vertex to the Directrix is $-p$. When p is positive, the parabola opens up or right. When p is negative, the parabola opens down or left.

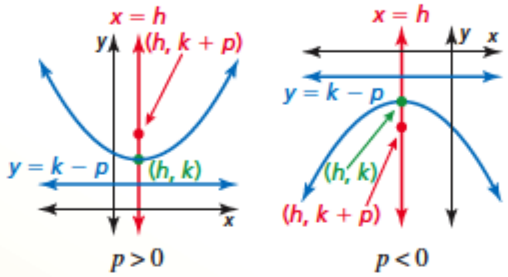
Standard Equations of a Parabola with Vertex at (h, k)

Vertical axis of symmetry $(x = h)$

Equation: $y = \frac{1}{4p}(x - h)^2 + k$

Focus: $(h, k + p)$

Directrix: $y = k - p$

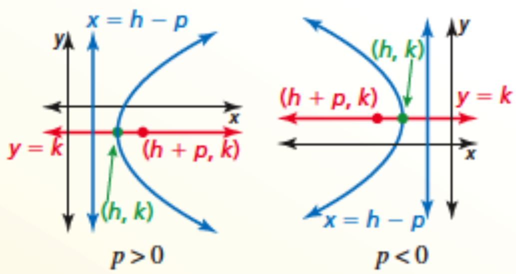


Horizontal axis of symmetry $(y = k)$

Equation: $x = \frac{1}{4p}(y - k)^2 + h$

Focus: $(h + p, k)$

Directrix: $x = h - p$



1. Use the distance formula to write an equation for the parabola with focus F(0, 2) and directrix y = -2.

Let P(x, y) be any point on the parabola.
Let D(x, -2) be the closest point on the Directrix to point P.
PD = PF

$$\sqrt{(x-x)^2+(y-(-2))^2} = \sqrt{(x-0)^2+(y-2)^2}$$
$$\sqrt{(y+2)^2} = \sqrt{x^2+(y-2)^2}$$
$$(y+2)^2 - x^2 + (y-2)^2$$
$$y^2+4y+4 = x^2+y^2-4y+4$$
$$8y = x^2$$
$$\boxed{y = \frac{1}{8}x^2}$$

2. Find the Focus, Directrix, and Axis of Symmetry of the parabola: $-4x = y^2$

Rewrite the equation in standard form. $x = -\frac{1}{4}y^2$ This parabola opens left.

Recall that the form should be $x = \frac{1}{4p}y^2$ so $\frac{1}{4p} = -\frac{1}{4}$ Cross Multiply: $4p = -4$ $\quad p = -1$

Since the Vertex is at (0, 0) and $p = -1$, the Focus is 1 unit left of the Vertex. $\boxed{\text{F(-1, 0)}}$.
The directrix is to the right of the Vertex 1 unit. The equation is $\boxed{x = 1}$.
The Axis of Symmetry is $\boxed{\text{y = 0 or the x-axis}}$.

Assignment 117: Page 72, #'s 3-10, 13-20.