Descartes Rule of Signs:

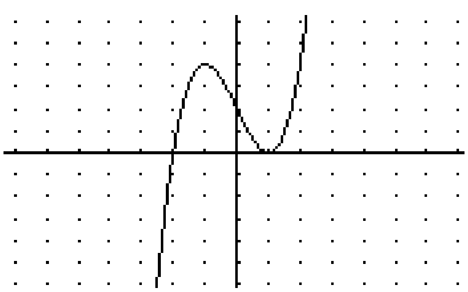
Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_2x^2 + a_1x + a_0$

be a polynomial with real coefficients and $a_0 \neq 0$.

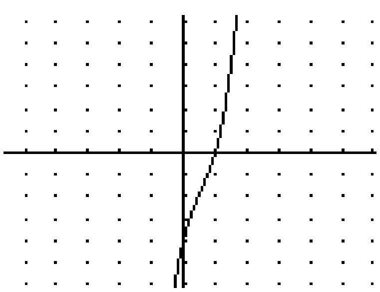
1. The number of **positive real zeros** of f is either equal to the number of variations in sign of f(x) or less than that number by an even integer.

2. The number of **negative real zeros** of f is either equal to the number of variations in sign of f(-x) or less than that number by an even integer.

ex1: $f(x) = x^3 - 3x + 2$ & $f(-x) = -x^3 + 3x + 2$
The number of sign changes of f(x) is 2. Therefore there will be 2 or 0 positive zeros.
The number of sign changes of f(-x) is 1. There there will be 1 negative zero.
Looking at the graph of f(x) we can verify the statements above.



ex2: $f(x) = 3x^3 - 5x^2 + 6x - 4$ & $f(-x) = -3x^3 - 5x^2 - 6x - 4$
The number of sign changes of f(x) is 3. Therefore there will be 3 or 1 positive zeros
The number of sign changes of f(-x) is 0. Therefore there will be no negative zeros
Looking at the graph of f(x) we can verify the statements above.



Upper and Lower Bound Rules:

Let f(x) be a polynomial with real coefficients and a positive leading coefficient.

Suppose f(x) is divided by (x - c), using synthetic division.

1. If each number in the bottom row have the **same sign** (zero can count as positive or negative) then c is **an upper bound** for the real zeros of f. In other words, no values greater than c will be a zero.

2. If each number in the bottom row have **alternating signs** (zero can count as positive or negative) then c is **a lower bound** for the real zeros of f. In other words, no values less than c will be a zero.

ex: $f(x) = 2x^4 + 3x^3 - 2x^2 + 5x + 8$

1	2	3	-2	5	8
		2	5	3	8
	2	5	3	8	16

-4	2	3	-2	5	8
		-8	20	-72	268
	2	-5	18	-67	276

Therefore 1 is an upper bound

Therefore -4 is a lower bound

Find the Zeros for: $f(x) = 3x^3 + 9x^2 - 12x - 36$

Candidates for Rational Zeros:

$\pm1, \pm2, \pm3, \pm4, \pm6, \pm9, \pm12, \pm18, \pm36, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}$

However, this is not the best approach.
We can factor out the 3 first

$3(x^3 + 3x^2 - 4x - 12)$. This gives a better choice of candidates $f(x) = 3\,g(x)$

$\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$ (No Fractions)

3	1	3	-4	-12
		3	18	42
	1	6	14	30

$g(3) = 30$

Don't check candidates higher than 3

All signs are the same

1	1	3	-4	-12
		1	4	0
	1	4	0	-12

$g(1) = -12$

There must be a zero between 1 and 3 by the IVT (Intermediate Value Theorem)

2	1	3	-4	-12
		2	10	12
	1	5	6	0

$g(2) = 0$

$f(x) = 3(x^3 + 3x^2 - 4x - 12) = 3(x - 2)(x^2 + 5x + 6) = 3(x - 2)(x + 3)(x + 2)$

All the zeros are: 2, -3, -2

Factor: $f(x) = 2x^4 - 7x^3 - x^2 + 15x - 9$

Candidates for Rational Zeros:

$\pm 1, \pm 3, \pm 9, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}$

-1	2	-7	-1	15	-9
		-2	9	-8	-7
	2	-9	8	7	-16

$f(-1) = -16$

3	2	-7	-1	15	-9
		6	-3	-12	9
	2	-1	-4	3	0

$f(3) = 0$ 3 is a zero

$f(x) = (x - 3)(2x^3 - x^2 - 4x + 3)$

1	2	-1	-4	3
		2	1	-3
	2	1	-3	0

$g(1) = 0$ 1 is a zero

At this point, we have: $(x - 3)(x - 1)(2x^2 + x - 3) = (x - 3)(x - 1)(2x + 3)(x - 1)$

All Zeros of f(x) are $3, -\frac{3}{2}$, and 1 with multiplicity 2

$f(x) = \boxed{(x - 1)^2(x - 3)(2x + 3)}$

Pre-Calc 1 Assignment 116 Monday, October 5, 2015 Hour   Name

Exer. 1-3: Use Descatre's Rule of Signs to determine the possible number of positive and negative zeros

1. $f(x) = 3x^4 - x^3 + 6x^2 - x + 5$

2. $f(x) = 3x^4 + 5x^3 - 6x^2 + 8x - 3$

3. $f(x) = x^3 + x^2 - 4x - 4$

Exer. 4-6: Show if the following functions have k as what type of bound.

4. $f(x) = x^4 - 4x^3 + 15$, $k = 3$

5. $f(x) = 2x^3 - 3x^2 - 12x + 8$, $k = -3$

6. $f(x) = 2x^4 - 8x + 3$, $k = 3$

7. $f(x) = 6x^3 + 5x^2 - 21x + 10$ Factor and find Zeros

8. Find the inverse of $y = 7x - 5$

9. Using Linear Regression, Predict y when x = 53
accurate to 5-decimal places.

x	1	3	4	8	12
y	6	8	14	15	19

10. $f(x) = 3x^2 - 12x + 1$ Write in Vertex Form. Give the Vertex. Find the Max or Min value of the function

11. $(2x^3 - 5x^2 + 4x - 6) \div (x + 3) =$

12. $(5x^3 + 2x^2 - 3x + 1) \div (x^2 - 2x - 1) =$

13. $f(x) = 3x^4 - 5x^3 + 2x + 24$ Use the Rational Zeros Test to List all Rational Candidates for Zeros.

14. $f(x) = x^4 - 4x^3 - 15x^2 + 58x - 40$ Factor and Find the Zeros.

15. $f(x) = 2x^2 - 5x - 4$ Find the Zeros.