

## 1. Factor Theorem:

Polynomial  $f(x)$  has a factor  $(x - c)$  iff  $f(c) = 0$ .

$$f(x) = -2x^4 + 7x^3 + 9x^2 - 22x + 8$$

Find  $P(4)$

$$\begin{array}{r|rrrrr} 4 & -2 & 7 & 9 & -22 & 8 \\ & & -8 & -4 & 20 & -8 \\ \hline & -2 & -1 & 5 & -2 & 0 \end{array}$$

$f(4) = 0$ , Therefore  $x - 4$  is a factor of  $f(x)$

$$\begin{aligned} f(x) &= 2x^4 + 7x^3 + 9x^2 - 22x + 8 \\ &= (2x^3 - x^2 + 5x - 2)(x - 4) + 0 \\ &= (2x^3 - x^2 + 5x - 2)(x - 4) \end{aligned}$$

## 2. Rational Zero Test:

If polynomial  $f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_2 x^2 + a_1 x + a_0$

has integer coefficients, every rational zero of  $f$  has the form  $\frac{p}{q}$

where  $p$  and  $q$  have no common factors other than 1,

then  $p$  is a factor of  $a_0$ , and  $q$  is a factor of  $a_n$ .

3. Find all the Rational Candidates for Zeros of  $f(x) = 6x^4 - 3x^3 + 2x^2 - 5x + 5$   
 All Factors of 5 are 1 and 5. These numbers make up possible  $\pm$  numerators.  
 All Factors of 6 are 1, 2, 3, and 6. These numbers make up possible  $\pm$  denominators.  
 Therefore, all possible Rational Candidates for Zeros are:

$$\boxed{\pm 1, \pm 5, \pm \frac{1}{2}, \pm \frac{5}{2}, \pm \frac{1}{3}, \pm \frac{5}{3}, \pm \frac{1}{6}, \pm \frac{5}{6}}$$

Find the Rational Zeros of  $f(x) = 2x^3 + 3x^2 - 8x + 3$

Possible Candidates come from  $\frac{\text{Factors of } 3 \cdots 1, 3}{\text{Factors of } 2 \cdots 1, 2} = \pm \frac{1}{1}, \pm \frac{3}{1}, \pm \frac{1}{2}, \pm \frac{3}{2} = \boxed{\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}}$

By synthetic substitution, we can show that  $x = 1$  is a zero

$$\begin{array}{r|rrrr} 1 & 2 & 3 & -8 & 3 \\ & & 2 & 5 & -3 \\ \hline & 2 & 5 & -3 & 0 \end{array}$$

This gives us:

$$f(x) = (x - 1)(2x^2 + 5x - 3)$$

$$= (x - 1)(2x - 1)(x + 3)$$

Zeros are:  $1, \frac{1}{2}, -3$

Find Zeros of  $f(x) = 10x^3 - 15x^2 - 16x + 12$

Possible Candidates:  $\frac{\text{Factors of } 12}{\text{Factors of } 10} = \frac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12}{\pm 1, \pm 2, \pm 5, \pm 10}$

Eventually we can find that 2 is a zero

$$\begin{array}{r|rrrr} 2 & 10 & -15 & -16 & 12 \\ & & 20 & 10 & 12 \\ \hline & 10 & 5 & -6 & 0 \end{array}$$

$$f(x) = (x - 2)(10x^2 + 5x - 6)$$

We must use the quadratic formula to find the other zeros

$$\text{They are: } \frac{-5 \pm \sqrt{25 - 4(10)(-6)}}{20} = \frac{-5 \pm \sqrt{265}}{20}$$

Therefore: All zeros are

$$2, \frac{-5 \pm \sqrt{265}}{20}$$

## 02.03 Real Zeros of Polynomial Functions

Exer.1-2: Use synthetic division to show that  $x$  is a solution of the third-degree polynomial equation, and use the result to factor the polynomial completely. List all the real zeros of the function.

1.  $x^3 - 7x + 6 = 0, x = 2$

2.  $x^3 - 28x - 48 = 0, x = -4$

Exer. 3-8: (a) verify the given factors of the function  $f$ , (b) find the remaining factors of  $f$ , (c) use your results to write the complete factorization of  $f$ , and (d) list all real zeros of  $f$ .

3.  $f(x) = 2x^3 + x^2 - 5x + 2, (x + 2)$

4.  $f(x) = 3x^3 + 2x^2 - 19x + 6, (x + 3)$

5.  $f(x) = x^4 - 4x^3 - 15x^2 + 58x - 40, (x - 5)(x + 4)$

6.  $f(x) = 8x^4 - 14x^3 - 71x^2 - 10x + 24, (x + 2), (x - 4)$

7.  $f(x) = 6x^3 + 41x^2 - 9x - 14, (2x + 1)$

8.  $f(x) = 2x^3 - x^2 - 10x + 5, (2x - 1)$

Exer. 9-10: Use the Rational Zeros Test to list all possible rational zeros of  $f$ .

9.  $f(x) = x^3 + 3x^2 - x - 3$

10.  $f(x) = 4x^5 - 8x^4 - 5x^3 + 10x^2 + x - 2$