PC Thursday, October 1 2015

Long Division:

1. $(4x^4 + 7x^3 - 4x^2 + 3x - 8) \div (x^2 + 2x - 1)$

$$
\begin{array}{r|l}
 & 4x^2 - x + 2 \\
\hline
x^2 + 2x - 1) & 4x^4 + 7x^3 - 4x^2 + 3x - 8 \\
 & \underline{4x^4 + 8x^3 - 4x^2} \\
 & -x^3 + 3x - 8 \\
 & \underline{-x^3 - 2x^2 + x} \\
 & 2x^2 + 2x - 8 \\
 & \underline{2x^2 + 4x - 2} \\
 & -2x - 6
\end{array}
$$

The Complete Answer to the Division: $\boxed{4x^2 - x + 2 + \frac{-2x - 6}{x^2 + 2x - 1}}$

2. $(2x^3 + 6x^2 - 15x + 28) \div (x + 5)$

$$
\begin{array}{r|l}
 & 2x^2 - 4x + 5 \\
\hline
x + 5) & 2x^3 + 6x^2 - 15x + 28 \\
 & \underline{2x^3 + 10x^2} \\
 & -4x^2 - 15x + 28 \\
 & \underline{-4x^2 - 20x} \\
 & 5x + 28 \\
 & \underline{5x + 25} \\
 & 3
\end{array}
$$

The Complete Answer to the Division: $\boxed{2x^2 - 4x + 5 + \frac{3}{x + 5}}$

3. Synthetic Division:

Consider a Dividend: $p(x) = 2x^3 + 6x^2 - 15x + 28$

Consider a Divisor: $d(x) = x + 5$

Divide $p(x) \div d(x)$

$(2x^3 + 6x^2 - 15x + 28) \div (x + 5)$

-5	2	6	-15	28
		-10	20	-25
	2	-4	5	3

These Numbers are the Coefficients of the DEPRESSED Polynomial portion of the Quotient along with the REMAINDER

Consider the Quotient without the Remainder: $q(x) = 2x^2 - 4x + 5$

Consider the Remainder: $r(x) = 3$

The Complete Answer is

$$\boxed{2x^2 - 4x + 5 + \frac{3}{x + 5}}$$

4. The Remainder Theorem:

We've seen several divisions that have answers that look like: $\frac{p(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$.

If we multiply both sides by d(x), we get: $p(x) = d(x)\, q(x) + r(x)$

Evaluate the above for a specific number such as x = a: $p(a) = d(a)\, q(a) + r(a)$

Suppose $d(a) = 0$, then that would make the equation: $p(a) = r(a)$ = The Remainder

So how do we make $d(a) = 0$? We let $d(a) = a + 5$ in the above problem, and let $a = -5$

Then: $p(-5) = r(-5)$. In the above problem: $p(-5) = 3$

Let's check this answer: $p(x) = 2x^3 + 6x^2 - 15x + 28$

So: $p(-5) = 2(-5)^3 + 6(-5)^2 – 15(-5) + 28 = 2(-125) + 6(25) – 15(-5) + 28 = -250 + 150 + 75 + 28 = 3$

Notice that this 3 is the last number in the Synthetic Division Set-Up.

We can say that $p(a) = r(a)$. We call this fact: $\boxed{\text{The Remainder Theorem}}$.

This set-up can be called $\boxed{\text{Synthetic Substitution}}$. It can be used to find p(-5)

5. Synthetic Substitution:

$f(x) = 2x^3 + 5x^2 - 13x + 22$. Find f(-2)

-2	2	5	-13	22
		-4	-2	30
	2	1	-15	52

Therefore: **f(-2) = 52**.

This is valid by the Remainder Theorem.

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