Related Rates

Write a relation involving the quantity whose rate of change is sought and the quantity of the item whose rate of change is accessible.
Use Implicit Differentiation, then solve for the desired rate of change.

1. The radius r of a sphere is increasing at a rate of 2 inches per minute.
   Find the rate of change of the volume when r = 6 inches

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\left.\frac{dV}{dt}\right|_{r=6} = 4\pi \bullet 36 \bullet 2$$

$$= \boxed{288\pi \text{ in}^3/\text{min}}$$

2. The radius, r, of a cone is changing at a constant rate of 2 in/min.
   The height, h, is 3 times the radius. Find the rate at which the volume, V, is changing when r = 6 inches

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi r^2(3r)$$

$$V = \pi r^3$$

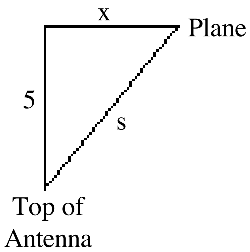
$$\frac{dV}{dt} = \pi\left(3r^2\right)\frac{dr}{dt}$$

$$\left.\frac{dV}{dt}\right|_{r=6} = \pi(108)(2)$$

$$\left.\frac{dV}{dt}\right|_{r=6} = \boxed{216\pi \text{ in}^3/\text{min}}$$

3. An airplane is flying at a constant altitude of 5 miles and passes directly over a radar antenna. When the plane is 10 miles from the antenna, the radar detects that this distance, s, is changing at a rate of 240 miles per hour. What is the horizontal speed of the plane?

Often a sketch of the situation can be helpful.



$s^2 = x^2 + 25$

When $s = 10$, $x^2 = 75$ □ $x = \sqrt{75} = 5\sqrt{3}$

$2s \frac{ds}{dt} = 2x \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{s}{x} \frac{ds}{dt}$

$\left.\frac{dx}{dt}\right|_{s=10} = \frac{10}{5\sqrt{3}} \bullet 240$

$\left.\frac{dx}{dt}\right|_{s=10} = \boxed{160\sqrt{3} \text{ mph}} \approx \boxed{277.128 \text{ mph}}$.

4. The radius of a sphere is increasing at a constant rate of 5 feet per minute. Find the rate at which the Volume is changing when the radius is 20 feet.

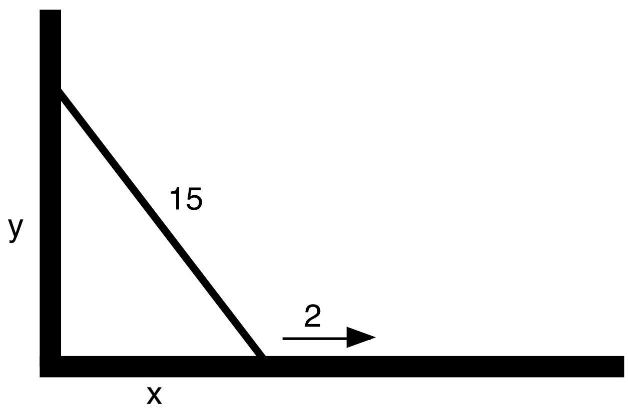
The Volume formula for a sphere is $V = \frac{4}{3} \pi r^3$.

Take the derivative with respect to time.

$\frac{dV}{dt} = \frac{4}{3} \pi \bullet 3r^2 \frac{dr}{dt}$

$\left.\frac{dV}{dt}\right|_{r=20} = \frac{4}{3} \pi \bullet 3(20)^2 \bullet 5 = \boxed{8000\pi \text{ ft}^3/\text{min}}$

5. A 15 foot ladder is leaning against a vertical wall. The bottom is being pulled away from the wall along the ground at a rate of 2 feet per minute. At what rate is the height of the ladder changing when the base is 9 feet from the base of the wall?



We have a Pythagorean Relation: $x^2 + y^2 = 225$

Take the derivative with respect to time.

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$2y\frac{dy}{dt} = -2x\frac{dx}{dt}$$

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

At the moment of interest,
We know x and $\frac{dx}{dt}$, but we need to know y

$y^2 = 225 - x^2 = 225 - 81 = 144$
$y = 12$

$$\left.\frac{dy}{dt}\right|_{x=9} = -\frac{9}{12}\cdot 2 = -\frac{3}{2}\,ft/\min$$

6. Sand is being poured onto a conical pile at the rate of $12\ \text{ft}^3/\text{min}$. The coefficient of friction of the sand is such that the height of the pile is twice the radius. Find the rate of change of the height when the radius is 5.

$$V = \frac{1}{3}\pi r^2 h \qquad \text{Also} \qquad h = 2r \ \square \ r = \frac{1}{2} h$$

$$V = \frac{1}{3}\pi\left(\frac{1}{2} h\right)^2 h = \frac{1}{12}\pi h^3$$

$$\frac{dV}{dt} = \frac{1}{12}\pi \bullet 3h^2 \frac{dh}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{4}{\pi h^2}\frac{dV}{dt}$$

$$\left.\frac{dh}{dt}\right|_{r=5} = \frac{4}{100\pi} \bullet 12 = \boxed{\frac{12}{25\pi}\ \text{ft/min}}$$

Assignment 114: Page 154, #'s 15, 18, 23, 28