

Find the zeros algebraically

1. $f(x) = 12x^3 + 26x^2 - 10x$

$$= 2x(6x^2 + 13x - 5)$$

$$= 2x(3x - 1)(2x + 5)$$

The Zeros are $x = 0, \frac{1}{3}, -\frac{5}{2}$

2. $f(x) = 15x^4 - 14x^2 - 8$

This example is not a Quadratic Function, but it is “Quadratic Like”.

If we define $w = x^2$, we can rewrite the function in terms of w.

$$f(w) = 15w^2 - 14w - 8$$

$$= (5w + 2)(3w - 4)$$

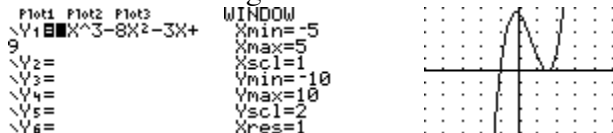
The Zeros in terms of w are $w = x^2 = -\frac{2}{5}, \frac{4}{3}$

Finding the legal square roots of w, we get $x = \pm \frac{2}{\sqrt{3}} = \pm \frac{2\sqrt{3}}{3}$

3. $f(x) = 4x^3 - 8x^2 - 3x + 9$

Since Factoring is not immediately obvious, we will use the calculator to find one of the zeros.

The window settings are also shown.



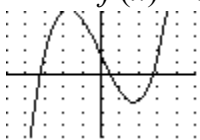
From the graph it appears that -1 is a zero. Therefore $(x + 1)$ is a factor.

So we do a division: $(4x^3 - 8x^2 - 3x + 9) \div (x + 1) = 4x^2 - 12x + 9$

So far we have: $(4x^3 - 8x^2 - 3x + 9) = (x + 1)(4x^2 - 12x + 9) = (x + 1)(2x - 3)^2$

The Zeros are $x = -1, \frac{3}{2}$. The $\frac{3}{2}$ has multiplicity two.

- 4 Use the Intermediate Value Theorem and a graphing utility to find graphically any intervals of length 1 in which $f(x) = 0.7x^2 - 6.4x + 3$ is guaranteed to have a zero.



From the graph there are several answers to choose from. They are $[-4, -3]$ or $[0, 1]$ or $[2, 3]$, because, for instance $f(-4) < 0$ and $f(-3) > 0$, so there is a zero between -4 and -3 by the IVT (Intermediate Value Theorem).

Exer. 1-6: Find the zeros algebraically.

1. $g(x) = 5x^2 - 10x - 5$

2. $y = \frac{1}{4}x^3(x^2 - 9)$

3. $g(t) = t^5 - 6t^3 + 9t$

4. $f(x) = 5x^4 + 15x^2 + 10$

5. $y = 4x^3 + 4x^2 - 7x + 2$

6. $y = x^5 - 5x^3 + 4x$

Exer. 7-10: Use the Intermediate Value Theorem and a graphing utility to find graphically any intervals of length 1 in which the polynomial function is guaranteed to have a zero.

7. $f(x) = 1.22x^3 - 3x^2 + 1$

8. $f(x) = 0.9x^2 + 4.5x - 3.4$

9. $f(x) = x^3 - 4x + 1$

10. $h(x) = -2x^3 - 6x + 3$