

Intermediate Value Theorem

Let a and b be real numbers such that $a < b$. If f is a polynomial function such that $f(a) \neq f(b)$, then in the interval $[a, b]$, f takes on every value between $f(a)$ and $f(b)$.

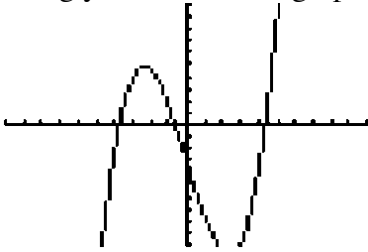
Example 1:

$$f(x) = 0.3x^3 - 5x - 3$$

Using a graphing utility, find all open intervals of length 1 that contain zeros of $f(x)$.

Solution:

Using your calculator, graph the function.



Notice the x -intercepts are between -4 & -3 , -1 & 0 , 4 & 5

The intervals are: $\boxed{(-4, -3), (-1, 0), (4, 5)}$.

Example 2:

Find the zeros algebraically: $g(x) = 5x^2 - 10x - 5$

Solution:

Since $g(x)$ cannot be factored, we use the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\begin{aligned} \text{So we get: } x &= \frac{10 \pm \sqrt{100 - 4(5)(-5)}}{10} = \frac{10 \pm \sqrt{100 + 100}}{10} = \frac{10 \pm \sqrt{200}}{10} = \frac{10 \pm \sqrt{2 \cdot 100}}{10} \\ &= \frac{10 \pm 10\sqrt{2}}{10} = \boxed{1 \pm \sqrt{2}}. \end{aligned}$$

Example 3:

Find the zeros algebraically: $f(x) = x^5 - 6x^3 + 9x$

Solution:

Factor Completely: $f(x) = x(x^4 - 6x^2 + 9) = x(x^2 - 3)(x^2 - 3)$

Therefore: $x = 0, x^2 = 3$ So the zeros are at $\boxed{x = 0 \text{ and } \pm\sqrt{3}}$.