PC Notes

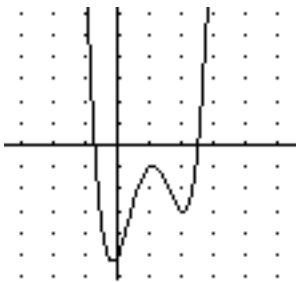## End Behavior - Leading Coefficient Test
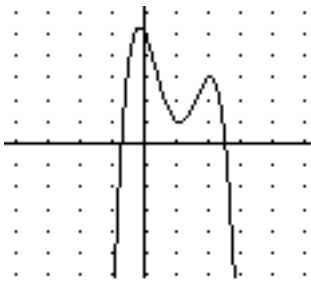
1. When the leading term is Positive and of Even Degree,
   then $\lim\limits_{x \to -\infty} f(x) \to +\infty$ and $\lim\limits_{x \to +\infty} f(x) \to +\infty$

2. When the leading term is Negative and of Even Degree,
   then $\lim\limits_{x \to -\infty} f(x) \to -\infty$ and $\lim\limits_{x \to +\infty} f(x) \to -\infty$

3. When the leading term is Positive and of Odd Degree,
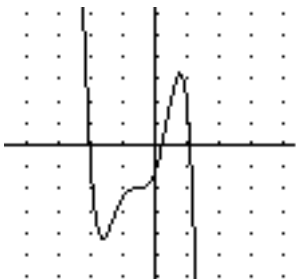   then $\lim\limits_{x \to -\infty} f(x) \to -\infty$ and $\lim\limits_{x \to +\infty} f(x) \to +\infty$
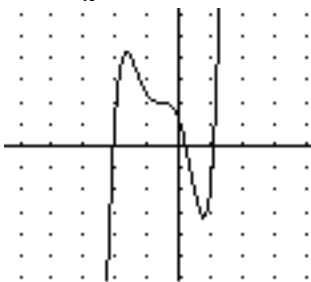
4. When the leading term is Negaive and of Odd Degree,
   then $\lim\limits_{x \to -\infty} f(x) \to +\infty$ and $\lim\limits_{x \to +\infty} f(x) \to -\infty$

Real Zeros of Polynomial Functions:
If f is a polynomial function and a is a real number, the following statements are equivalent.
1. $x = a$ is a zero
2. $x = a$ is a solution of the equation $f(x) = 0$
3. $(x - a)$ is a factor of $f(x)$
4. $(a, 0)$ is an x-intercept of the graph of f.

Find all zeros of $f(x) = x^3 - x^2 - 2x$
$f(x) = x^3 - x^2 - 2x = x(x^2 - x - 2) = x(x - 2)(x + 1)$
The zeros are $0, 2, -1$

Find all zeros of $f(x) = -2x^4 + 2x^2$
$f(x) = -2x^4 + 2x^2 = -2x(x^2 - 1) = -2x(x + 1)(x - 1) = 0$
The zeros are $0, -1, 1$

Repeated zeros → Multiplicity
For a polynomial function, a factor of $(x - a)^k$, yields a repeated zero $x = a$ of multiplicity k.
1.   If k is odd, the graph crosses the x-axis at $x = a$.
2.   If k is even, the graph touches the x-axis at $x = a$, but does not cross the x-axis there.

Find all real zeros of $f(x) = x^5 - 3x^3 - x^2 - 4x - 1$
Since we have not learned to factor a quintic (5th degree Polynomial), we can use a graping calculator.
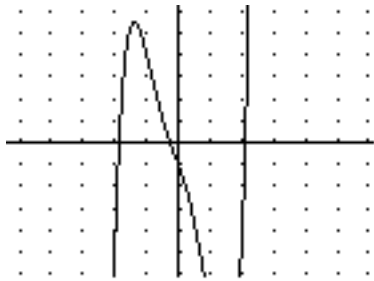
Using the Calculate Function, we find approximate zeros as: -1.86081, -0.254102, 2.11491

Find a polynomial function with zeros: $-\frac{1}{2}$, $3, 3$

The obvious factors are $\left(x + \frac{1}{2}\right)$, $(x - 3), (x - 3)$
To avoid fractions we will use $(2x + 1), (x - 3), (x - 3)$
$f(x) = (2x + 1)(x - 3)(x - 3)$
$f(x) = (2x + 1)(x^2 - 6x + 9)$
$f(x) = 2x^3 - 12x^2 + 18x + x^2 - 6x + 9$
$f(x) = 2x^3 - 11x^2 + 12x + 9$

Find a polnomial with zeros: $3, 2 + \sqrt{11}$, $2 - \sqrt{11}$
$f(x) = (x - 3)\left[x - \left(2 + \sqrt{11}\right)\right]\left[x - \left(2 - \sqrt{11}\right)\right]$
$f(x) = (x - 3)\left[(x - 2) - \sqrt{11}\right]\left[(x - 2) + \sqrt{11}\right]$
$f(x) = (x - 3)\left[(x - 2)^2 - 11\right]$
$f(x) = (x - 3)(x^2 - 4x + 4 - 11)$
$f(x) = (x - 3)(x^2 - 4x - 7)$
$f(x) = x^3 - 4x^2 - 7x - 3x^2 + 12x + 21$

$f(x) = x^3 - 7x^2 + 5x + 21$

Use the calculating feature to find the approximate values for the zeros.
Approximate Zeros are: -3.740779, -0.6138804, 4.3546593

For the following Graphs:
a.   Is the Leading Coefficient Negative or Positive
b.   Is the Degree of f(x) Odd or Even
c.   List all of the Variable Factors
d.   Describe the left and right-hand end behavior of the graph of the polynomial function
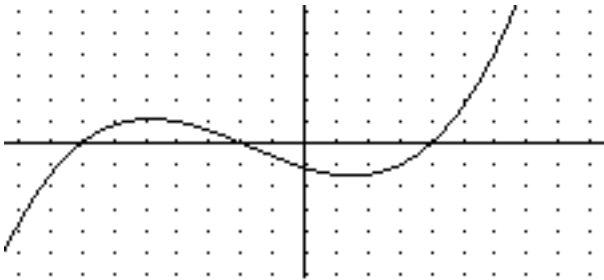
1.
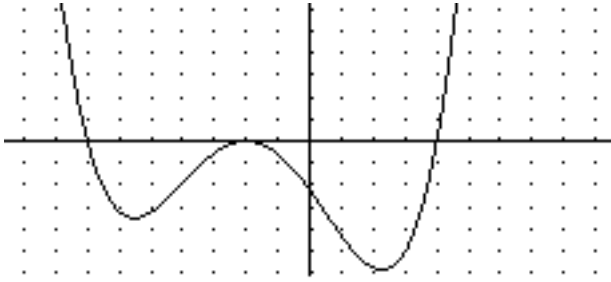
Positive - Even - $(x + 7)$ $(x + 2)^2$ $(x - 4)$

$\lim\limits_{x \to -\infty} f(x)$ ∞ and $\lim\limits_{x \to \infty} f(x)$ ∞

2.

Positive - Odd - $(x + 7)$ $(x + 2)$ $(x - 4)$

$\lim\limits_{x \to -\infty} f(x)$ $-\infty$ and $\lim\limits_{x \to \infty} f(x)$ ∞

3.
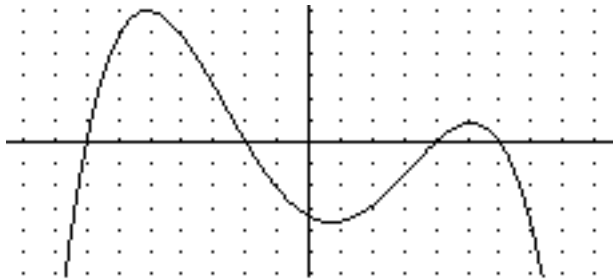
Negative - Even - (x + 7) (x + 2) (x - 4) (x - 6)

$\lim\limits_{x \to -\infty} f(x)$ -∞ and $\lim\limits_{x \to \infty} f(x)$ -∞

4.
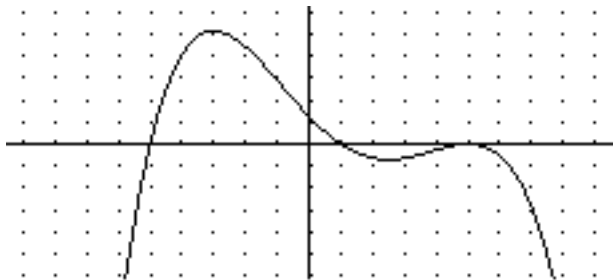
Negative - Even - (x + 5) (x - 1) (x - 5)^2

$\lim\limits_{x \to -\infty} f(x)$ -∞ and $\lim\limits_{x \to \infty} f(x)$ -∞

5.
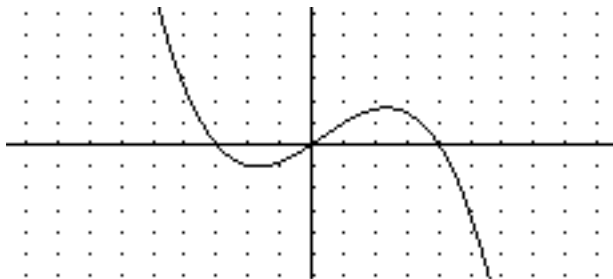
Negative - Odd - (x + 3) x (x - 4)

$\lim\limits_{x \to -\infty} f(x)$ ∞ and $\lim\limits_{x \to \infty} f(x)$ -∞

6.
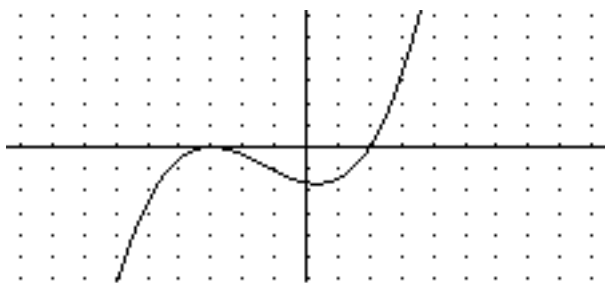
$\boxed{\text{Positive - Odd - } (x + 3)^2 \ (x - 2)}$

$\lim\limits_{x \to -\infty} f(x) \quad -\infty \ \text{ and } \ \lim\limits_{x \to \infty} f(x) \quad \infty$

7.
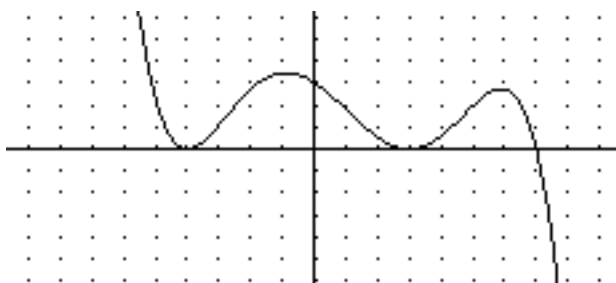
$\boxed{\text{Negative - Odd - } (x + 4)^2 \ (x - 3)^2 \ (x - 7)}$

$\lim\limits_{x \to -\infty} f(x) \quad \infty \ \text{ and } \ \lim\limits_{x \to \infty} f(x) \quad -\infty$

Assignment 111

02.02 Polynomial Functions of Higher Degree
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