Quadratic Functions

Definition:
A quadratic function is any function that may be written in the form $f(x) = ax^2 + bx + c$ where a, b, and c are real numbers, but $a \neq 0$.

When a is **positive**, the graph opens **upward**.
When a is **negative**, then the graph opens **downward**.

Examples:

1.	$f(x) = 2x^2 - 5$	$a = 2, b = 0, c = -5$
2.	$g(x) = 4x - 7x^2 + 9$	$a = -7, b = 4, c = 9$
3.	$k(x) = 5x^3 - 4x$	Not a Quadratic Function
4.	$y = -5x^2$	$a = -5, b = 0, c = 0$

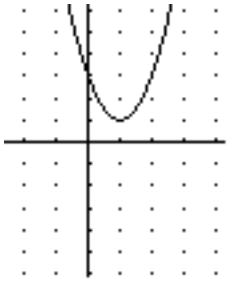
Forms of Quadratic Functions

1.	General Form:	$f(x) = ax^2 + bx + c$	a gives direction of opening, c gives y-intercept
2.	Standard or Vertex Form:	$f(x) = a(x - h)^2 + k$	(h, k) gives coordinates of Vertex
3.	Intercept Form:	$f(x) = a(x - p)(x - q)$	p and q are the x-intercepts

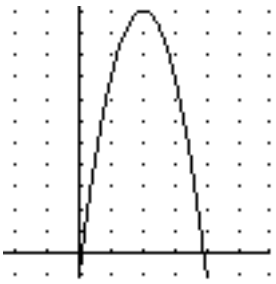
Convert a General Form to a Vertex Form by "Completing The Square" ... Then locate the vertex.

1. $f(x) = 2x^2 - 4x + 3$
    1. Factor the constant, a, from the x^2 and x terms.
       $f(x) = 2(x^2 - 2x) + 3$
    2. Find half the coefficient of the x term and square the result
       Place this answer in the parentheses
       Also place the opposite times a outside
       $f(x) = 2(x^2 - 2x + 1) + 3 - 2$
    3. Factor the parentheses and combine the constants

       Vertex Form: $\boxed{f(x) = 2(x - 1)^2 + 1}$ → Vertex: $\boxed{(1, 1)}$



2. $f(x) = -3x^2 + 12x - 1$
   $f(x) = -3(x^2 - 4x) - 1$
   $f(x) = -3(x^2 - 4x + 4) - 1 + 12$
   $\boxed{f(x) = -3(x - 2)^2 + 11}$ $\boxed{V(2, 11)}$

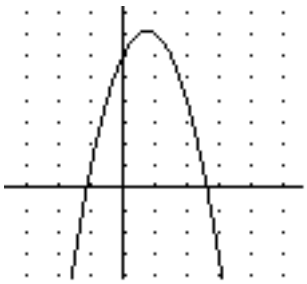


3. $f(x) = -2x^2 + 3x + 6$

$f(x) = -2\left(x^2 - \frac{3}{2}x\right) + 6$

$f(x) = -2\left(x^2 - \frac{3}{2}x + \frac{9}{16}\right) + 6 + \frac{9}{8}$

$\boxed{f(x) = -2\left(x - \frac{3}{4}\right)^2 + \frac{57}{8}}$ $\boxed{V\left(\frac{3}{4}, \frac{57}{8}\right)}$



4. $f(x) = ax^2 + bx + c$

$f(x) = a\left(x^2 + \frac{b}{a}x\right) + c$

$f(x) = a\left(x^2 + \frac{b}{a} + \frac{b^2}{4a^2}\right) + \left(c - \frac{b^2}{4a}\right)$

$\boxed{f(x) = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}}$ $\boxed{V\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)}$

The Last Example gives us a quick way to find the vertex.

The x-coordinate of the vertex is $-\frac{b}{2a}$

Find the x-coordinate of the vertex

1. $f(x) = 7x^2 + 6x - 1$

$a = 7, b = 6$

$-\frac{b}{2a} = -\frac{6}{14} = \boxed{-\frac{3}{7}}$

2. $f(x) = 2x^2 - 12x + 5$

$a = 2, b = -12$

$-\frac{b}{2a} = \boxed{3}$

The y-coordinate of the vertex gives the maximum value if the graph opens downward.
The y-coordinate of the vertex gives the minimum value if the graph opens upward

1. Find the maximum value of $f(x) = -2x^2 - 12x - 3$

   $c - \frac{b^2}{4a} = -3 + \frac{144}{-8} = -3 + 18 = \boxed{15}$

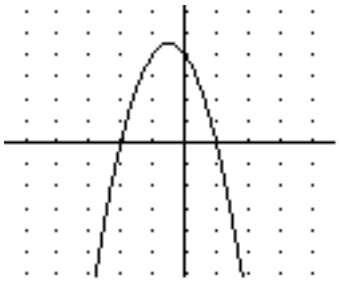
2. Write a quadratic function whose graph has x-intercepts 1 and -2 in general form.

   $y = a(x - 1)(x + 2)$, where a is any number we choose

   $y = -2(x - 1)(x + 2)$

   $y = -2(x^2 + x - 2)$

   $\boxed{y = -2x^2 - 2x + 4}$



To find x-intercepts (also called zeros), let $y = 0$, then solve for x.

Assignment 110
02.01 Quadratic Functions
Page 99, #'s 1-52 Multiples of 4