Notes

Derivative of ln(x) and Derivative of e^x .

Recall:

1. Discrete Interest Formula: $A = P\left(1 + \frac{r}{n}\right)^{nt}$
2. Continuous Interest Formula: $A = Pe^{rt}$
3. $\lim_{n\to\infty} P\left(1 + \frac{r}{n}\right)^{nt} = Pe^{rt}$

   $\lim_{n\to\infty} \left(1 + \frac{r}{n}\right)^{nt} = e^{rt}$

   $\lim_{n\to\infty} \left(1 + \frac{r}{n}\right)^{n} = e^{r}$

4. Using a calculator we can find an approximation for e ≈ 2.718281828
   Memorize this 10-digit approximation.

5. Find the derivative of ln x

$$\frac{d}{dx}\ln x = \lim_{\Delta x \to 0} \frac{\ln(x+\Delta x) - \ln(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\ln\left(\frac{x+\Delta x}{x}\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x}\ln\left(1+\frac{\Delta x}{x}\right)$$

$$= \lim_{\Delta x \to 0} \ln\left(1+\frac{\Delta x}{x}\right)^{1/\Delta x}$$

$$= \lim_{1/\Delta x \to \infty} \ln\left(1+\frac{1/x}{1/\Delta x}\right)^{1/\Delta x}$$

$$= \ln e^{1/x}$$

$$= \frac{1}{x}\ln e$$

$$= \frac{1}{x}$$

$$\boxed{\frac{d}{dx}\ln x = \frac{1}{x}}.$$

We will now find the derivative of $f(x) = e^x$

$$\frac{d}{dx} \ln e^x = \frac{1}{e^x} \frac{d}{dx} e^x \quad \text{By the Chain Rule}$$

ALSO

$$\frac{d}{dx} \ln e^x = \frac{d}{dx} x \ln e = \frac{d}{dx} x = 1$$

THEREFORE

$$\frac{1}{e^x} \frac{d}{dx} e^x = 1$$

$$\boxed{\frac{d}{dx} e^x = e^x}$$

<u>Implicit Differentiation</u> is simply an application of the chain rule.

Find dy/dx by implicit differentiation.
$x^{1/2} + y^{1/2} = 9$

Taking the derivative **implicitly** term by term with respect to x:

$$\frac{1}{2} x^{-1/2} + \frac{1}{2} y^{-1/2} \bullet \frac{dy}{dx} = 0$$

$$\frac{1}{2} y^{-1/2} \bullet \frac{dy}{dx} = -\frac{1}{2} x^{-1/2}$$

$$\frac{dy}{dx} = \frac{-\frac{1}{2} x^{-1/2}}{\frac{1}{2} y^{-1/2}}$$

$$\frac{dy}{dx} = -\frac{x^{-1/2}}{y^{-1/2}}$$

$$\frac{dy}{dx} = -\frac{y^{1/2}}{x^{1/2}}$$

$$\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$$

$$\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}} \bullet \frac{\sqrt{x}}{\sqrt{x}}$$

$$\frac{dy}{dx} = -\frac{\sqrt{xy}}{x}$$

Given: $\cos x = 3xy + 2x^2 - \sin y$

Find $\frac{dy}{dx}$

$$-\sin x = 3\left(x\frac{dy}{dx} + y\right) + 4x - \cos y\left(\frac{dy}{dx}\right)$$

$$-\sin x = 3x\frac{dy}{dx} + 3y + 4x - \cos y\left(\frac{dy}{dx}\right)$$

$$-\sin x = \frac{dy}{dx}(3x - \cos y) + 3y + 4x$$

$$\boxed{\frac{dy}{dx} = \frac{-\sin x - 4x - 3y}{3x - \cos y}}$$

More Implicit Differentiation:

This allows us to make use of the chain rule to take the derivative of several functions with respect to any variable.

Given a cone where the height, h, is twice the radius, r. The radius is increasing at a constant rate of 6 meters per second. Find the rate at which the volume is changing when the radius is 2 meters.

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi r^2(2r)$$

$$V = \frac{1}{3}\pi(2r^3)$$

$$V = \frac{2}{3}\pi r^3$$

We now take the derivative of each term with respect to t (time)

$$\frac{dV}{dt} = \frac{2}{3}\pi(3r^2)\frac{dr}{dt}$$

Let r = 2

$$\left.\frac{dV}{dt}\right|_{r=2} = \frac{2}{3}\pi(3 \bullet 4)(6)$$

$$\left.\frac{dV}{dt}\right|_{r=2} = \boxed{48\pi \frac{\text{meters}^2}{\text{second}}}$$

Assignment 110

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