Theorem 02.07 The Product Rule: The product of two differentiable functions f and g is itself differentiable. Moreover, the derivative of fg is the first function times the derivative of the second, plus the second function times the derivative of the first.

$$\frac{d}{dx}[f(x)\,g(x)] = f(x)\,g'(x) + g(x)\,f'(x)$$

Proof: $$\lim_{\Delta x \to 0} \frac{f(x+\Delta x)\,g(x+\Delta x) - f(x)\,g(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(x+\Delta x)\,g(x+\Delta x) - f(x+\Delta x)\,g(x) + f(x+\Delta x)\,g(x) - f(x)\,g(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left[f(x+\Delta x)\,\frac{g(x+\Delta x) - g(x)}{\Delta x} + g(x)\,\frac{f(x+\Delta x) - f(x)}{\Delta x} \right]$$

$$= \lim_{\Delta x \to 0} \left[f(x+\Delta x)\,\frac{g(x+\Delta x) - g(x)}{\Delta x} \right] + \lim_{\Delta x \to 0} \left[g(x)\,\frac{f(x+\Delta x) - f(x)}{\Delta x} \right]$$

$$= \lim_{\Delta x \to 0} f(x+\Delta x) \bullet \lim_{\Delta x \to 0} \frac{g(x+\Delta x) - g(x)}{\Delta x} + \lim_{\Delta x \to 0} g(x) \bullet \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \boxed{f(x)\,g'(x) + g(x)\,f'(x)}$$

Theorem 02.08 The Quotient Rule: The quotient f/g of two differentiable functions f and g is itself differentiable at all values of x for which g(x) $\neq 0$. Moreover, the derivative of f/g is given by the denominator times the derivative of the numerator minus the numerator times the derivative of the denominator, all divided by the square of the denominator.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\,f'(x) - f(x)\,g'(x)}{[g(x)]^2}$$

Proof: $$\lim_{x \to 0} \frac{\dfrac{f(x+\Delta x)}{g(x+\Delta x)} - \dfrac{f(x)}{g(x)}}{\Delta x} = \lim_{x \to 0} \frac{f(x+\Delta x)\,g(x) - f(x)\,g(x+\Delta x)}{\Delta x\, g(x+\Delta x)\,g(x)}$$

$$= \lim_{x \to 0} \frac{f(x+\Delta x)\,g(x) - f(x)\,g(x) + f(x)\,g(x) - f(x)\,g(x+\Delta x)}{\Delta x\, g(x+\Delta x)\,g(x)}$$

$$= \frac{\lim\limits_{\Delta x \to 0} \dfrac{g(x)[f(x+\Delta x) - f(x)]}{\Delta x} - \lim\limits_{\Delta x \to 0} \dfrac{f(x)[g(x+\Delta x) - g(x)]}{\Delta x}}{\lim\limits_{\Delta x \to 0} [g(x+\Delta x)\,g(x)]}$$

$$= \frac{g(x)\left[\lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x}\right] - f(x)\left[\lim\limits_{\Delta x \to 0} \dfrac{g(x+\Delta x) - g(x)}{\Delta x}\right]}{\lim\limits_{\Delta x \to 0} [g(x+\Delta x)\,g(x)]}$$

$$= \boxed{\frac{g(x)\,f'(x) - f(x)\,g'(x)}{[g(x)]^2}}$$

A Phrase that may help us remember is:
"Low D High Minus High D Low all over Low Low"

The rate of change of Position is Velocity.
The rate of change of Velocity is Acceleration
The rate of change of Acceleration is Jerk
Speed is the Absolute Value of Velocity

An object dropped from a height of 100 feet moves by the position function, $s(t) = -16t^2 + 100$, where t is time in seconds, and s is position height in feet.

a. Find the **average velocity** on the interval [1, 2].

$s(2) = 36$, $s(1) = 84$, so $\Delta s = 36 - 84$. $\Delta t = 2 - 1 = 1$.

$\frac{\Delta s}{\Delta t} = \boxed{\text{-48 feet per second}}$

b. Find the **instantaneous velocity** at x = 2.

$s'(t) = -32t$ □ $-32(2) = \boxed{\text{-64 feet per second}}$

c. Find the **speed** at x = 2.

The **Speed** of an object is the absolute value of the velocity. Therefore, speed cannot be negative.

The speed = $\boxed{\text{64 feet per second}}$

Assignment 108
Page 115, #'s 10, 18, 19-24, 31, 33, 35, 37