

Systems of Linear Equations

Tickets for a concert sell for \$75 in Section A, \$55 in Section B, and \$30 in Section C. There are 3 times as many seats in Section B as in Section A. There are 23,000 seats sold. The income from selling all of the seats is \$870,000.

This information can give us a System of 3 Linear Equations.

Let x = The Number of seats in Section A

Let y = The Number of seats in Section B

Let z = The Number of seats in Section C

From the first Statement: $75x + 55y + 30z$ Represents all money collected

From the second Statement: $3x = y$

From the third Statement: $x + y + z = 23,000$

From the fourth Statement along with the first Statement: $75x + 55y + 30z = 870,000$

This gives us a system of equations:

$$\begin{cases} 3x = y \\ x + y + z = 2300 \\ 75x + 55y + 30z = 870,000 \end{cases}$$

We will start by substituting $3x$ for each instance of y in equations 2 and 3.

$$\begin{cases} x + 3x + z = 23,000 \\ 75x + 55(3x) + 30z = 870,000 \end{cases} \rightarrow \begin{cases} 4x + z = -23,000 \\ 240x + 30z = 870,000 \end{cases}$$

Next We Multiply the top Equation by -30 to get opposite z terms, then Add the equations.

$$\begin{cases} -120x - 30z = -690,000 \\ 240x + 30z = 870,000 \end{cases} \rightarrow -120x = 180,000 \rightarrow x = 1500 \rightarrow y = 4500 \rightarrow z = 17,000$$

The Solution:

There are 1500 Seats in Section A

There are 4500 Seats in Section B

There are 17,000 Seats in Section C

Solve the System:

$$\begin{cases} x + 4y - 6z = -1 \\ 2x - y + 2z = -7 \\ -x + 2y - 4z = 5 \end{cases}$$

Find a way to make x-terms cancel, or y-terms cancel, or z-terms cancel

If we multiply the top by -2 and the bottom by 2, then we can eliminate x from top and bottom, using the 2nd equation.

$$\begin{cases} -2x - 8y + 12z = 2 \\ 2x - y + 2z = -7 \quad \rightarrow \\ -2x + 4y - 8z = 10 \end{cases}$$

After Adding Equation 1 + Equation 2 and also Adding Equation 2 + Equation 3, we get:

$$\begin{cases} -9y + 14z = -5 \quad \rightarrow \\ 3y - 6z = 3 \end{cases}$$

Multiply Equation 2 by 3 to get opposite y-terms:

$$\begin{cases} -9y + 14z = -5 \quad \rightarrow \\ 9y - 18z = 9 \end{cases}$$

Add Equations 1 + Equation 2:

$$-4z = 4 \quad \rightarrow \quad z = -1$$

Find an Equation from above that has z and y to solve for y:

$$3y - 6z = 3 \quad \rightarrow \quad 3y - 6(-1) = 3 \quad \rightarrow \quad 3y + 6 = 3 \quad \rightarrow \quad y = -1$$

Find an Equation from above that has x and solve for it:

$$-x + 2y - 4z = 5 \quad \rightarrow \quad -x + 2(-1) - 4(-1) = 5 \quad -x - 2 + 4 = 5 \quad \rightarrow \quad -x + 2 = 5 \quad \rightarrow \quad x = 3$$

The ordered triple is (3, -1, -1).

Sometimes a System can have multiple solutions.

$$\begin{cases} 7x + 8y - 6z = 31 \\ -2x - 3y + z = -6 \\ x + y - z = 5 \end{cases}$$

Multiply the second equation by 6 and the 3rd by -6 to get opposite z terms, then Add Equation 1 + Equation 2 and also Add & Equation 1 + Equation 3.

$$\begin{cases} 7x + 8y - 6z = 31 \\ -12x - 18y + 6z = -36 \\ -6x - 6y + 6z = -30 \end{cases} \rightarrow \begin{cases} 7x + 8y - 6z = 31 \\ -5x - 10y = -6 \\ x + 2y = 1 \end{cases}$$

Now, Multiply Equation 3 by 5 then Add Equation 2 + Equation 3.

$$\begin{cases} 7x + 8y - 6z = 31 \\ -5x - 10y = -6 \\ 5x + 10y = 5 \end{cases}$$

We notice that our assumption that there is a unique answer leads to a contradiction that $0 = -1$

Therefore we can let y stay as y, and $x = -2y + 1$.

Looking at the top Equation, we can get $6z = 7x + 8y - 31$.

With our Substitutions we get: $6z = 7(-2y + 1) + 8y - 31 = -14y + 7 + 8y - 31 = -6y - 24$

Therefore $z = -y - 4$

The Ordered Triple: $(-2y + 1, y, -y - 4)$

Assignment 008

01.04 Solving Linear Systems – Page 34, #'s 3, 5, 6, 20, 25, 26.

Solve each system of equations.

$$3. \begin{cases} x + y - 2z = 5 \\ -x + 2y + z = 2 \\ 2x + 3y - z = 9 \end{cases}$$

$$5. \begin{cases} 2x + y - z = 9 \\ -x + 2y + z = -17 \\ 5x + 7y + z = 4 \end{cases}$$

$$6. \begin{cases} 3x + 2y - z = 8 \\ -3x + 4y + 5z = -14 \\ x - 3y + 4z = -14 \end{cases}$$

$$20. \begin{cases} x - 6y - 2z = -8 \\ -x + 5y + 3z = 2 \\ 3x - 2y - 4z = 18 \end{cases}$$

$$25. \begin{cases} x + y - z = 4 \\ 3x + 2y + 4z = 17 \\ -x + 5y + z = 8 \end{cases}$$

$$26. \begin{cases} 2x - y - z = 15 \\ 4x + 5y + 2z = 10 \\ -x - 4y + 3z = -20 \end{cases}$$