One-to-One (1-1) Function:
Function f is One-to-One Iff $f(a) = f(b) \rightarrow a = b$ for all a and b in the domain of f.

Although this is **not to be used in a proof**, 1-1 functions pass the horizontal line test.
That is, you cannot find a horizontal line that passes through the graph at more than one point.

$y = 3x - 1$ is a one-to-one function.
A small partial set of the elements it contains: $\{(1, 2), (-2, -7), (10, 29), (\sqrt{2}, 3\sqrt{2} - 1)\}$
plus an infinite number of other elements.

If $y(x) = y(10)$ which is 29, then x must equal 10

$y = x^2 - 6x + 13$ is a function, but it is not one-to-one.

For example $y(0) = y(6) = 13$, but $0 \neq 6$

To **prove** that a function **<u>IS</u>** one-to-one, we start by stating that:
$f(a) = f(b)$. Then we must show that as a result, we can arrive at $a = b$

To **prove** that a function **<u>IS NOT</u>** one-to-one, we must find a **<u>specific counter-example</u>**
that defies the definition.

Prove or Disprove that: $f(x) = 5x + 2$ is 1-1.

Let $f(a) = f(b)$

$5a + 2 = 5b + 2$	Definition of f
$5a = 5b$	Subtraction of 2
$a = b$	Divide by 5

Therefore, f(x) is one-to-one because $f(a) = f(b) \rightarrow a = b$
Q.E.D.

Prove or Disprove that: $f(x) = 2x^2 + 3$ is 1-1.

$f(4) = f(-4) = 35$, but $4 \neq -4$
Therefore, f(x) is not one-to-one.
Q.E.D.

Inverse Functions:
If $(f \circ g)(x) = f(g(x)) = x$, $\forall$ x in the domain of $(f \circ g)$ and
If $(g \circ f)(x) = g(f(x)) = x$, $\forall$ x in the domain of $(g \circ f)$,
Then f and g are **inverse** functions of each other.
$f^{-1}(x) = g(x)$ and $g^{-1}(x) = f(x)$
$f^{-1}(x)$ means **the inverse of f(x)**

Consider the following functions
G = {(2, 6), (-3, 4), (5, 1), (3, 2), (0, -1)} and
F = {(6, 2), (4, -3), (1, 5), (2, 3), (-1, 0)}

Note that both G and F are functions.
Also note that the reversal of each ordered pair in G gives an ordered pair in F.
When this is the case, and both are functions, then they are inverses of each other.
$G(a) = b \leftrightarrow F(b) = a$

A function has an inverse function iff it is one-to-one.

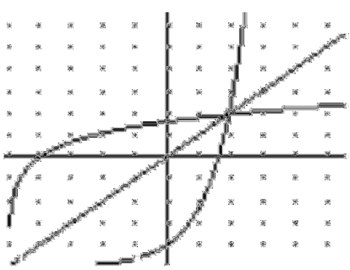
To Find the inverse of a one-to-one function, y = f(x) algebraically, exchange the x and y (or f(x)), then solve for the new y (or f(x)).

Find the inverse of $y = f(x) = 5x + 4$

1. Put x in place of all y and put y in place of all x: $x = 5y + 4$
2. Solve for this new y: $5y = x - 4$ $y = \frac{x-4}{5}$

$f(x) = x^2$ is not one-to-one, and therefore has no inverse.
f(x) contains elements: (2, 4) and (-2, 4). If it was a 1-1 function, then the inverse $f^{-1}(x)$ must contain (4, 2) and (4, -2) and therefore cannot be a function.

The graphs of two functions that are inverses of each other are symmetric with respect to the line y = x (The 45° Line)..



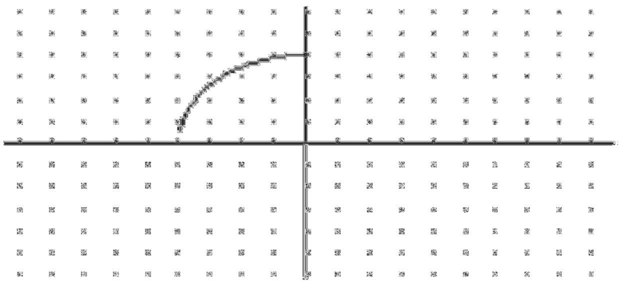
The graphs above represent two functions that are inverses of each other.

Find the inverse function of
$y = f(x) = \sqrt{16 - x^2}$, $-4 \le x \le 0$

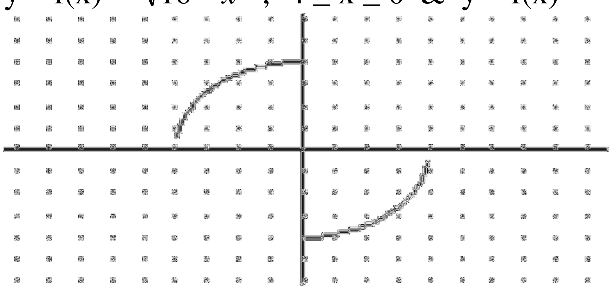
Rewrite:
$x = \sqrt{16 - y^2}$, $-4 \le y \le 0$ and $0 \le x \le 4$

Now Solve for y:
$x^2 = 16 - y^2$
$y^2 = 16 - x^2$
$y = f(x) = -\sqrt{16 - x^2}$, $0 \le x \le 4$

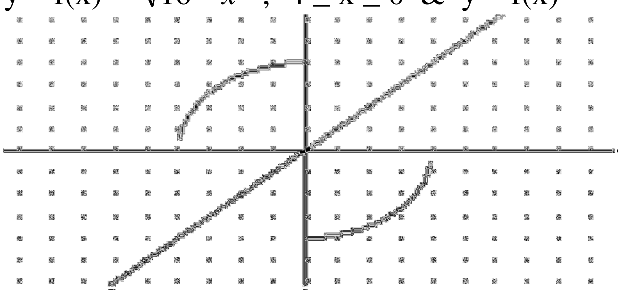
$y = f(x) = \sqrt{16 - x^2}$, $-4 \le x \le 0$



$y = f(x) = \sqrt{16 - x^2}$, $-4 \le x \le 0$ & $y = f(x) = -\sqrt{16 - x^2}$, $0 \le x \le 4$



$y = f(x) = \sqrt{16 - x^2}$, $-4 \le x \le 0$ & $y = f(x) = -\sqrt{16 - x^2}$, $0 \le x \le 4$



Assignment 007 Page 69, #'s 1, 3, 6, 9, 13, 14, 16, 18, 22, 24, 30, 31, 38, 52, 65, 82

Exer. 1-8: Find the inverse function of f informlly. Verify that $f(f^{-1}(x)) = x$.

1. $f(x) = 6x$

3. $f(x) = x + 7$

6. $f(x) = \frac{x - 1}{4}$

Exer. 9-14: (a) Show that f and g are inverse functions algebraically and (b) use a graphing utility to create a table of values for each function to numerically show that f and g are inverse functions.

9. $f(x) = -\frac{7}{2} x - 3$, $g(x) = -\frac{2x + 6}{7}$

13. $f(x) = -\sqrt{x - 8}$, $g(x) = 8 + x^2$, $x \leq 0$

14. $f(x) = \sqrt[3]{3x - 10}$, $g(x) = \frac{x^3 + 10}{3}$

Exer. 15-20: Show that f and g are inverse functions algebraically. Use a graphing utility to graph f and g in the same viewing window. Describe the relationship between the graphs.

16. $f(x) = \frac{1}{x}$, $g(x) = \frac{1}{x}$

18. $f(x) = 9 - x^2$, $x \geq 0$, $g(x) = \sqrt{9 - x}$

Exer. 21-24: Match the graph of the function with the graph of its inverse function. [The graphs of the inverse functions are labeled (a), (b), (c), and (d)]

22.

24.

Exer. 29-34: Determine if the graph is that of a function. If so, determine if the function is one-to-one.

30.

31.

Exer. 35-46: Use a graphing utility to graph the function and use the Horizontal Line Test to determine whether the function is one-to-one and so an inverse function exists.

38. $g(x) = \frac{4 - x}{6x^2}$

Exer. 47-58: Determine algebraically whether the function is one-to-one. Verify your answer graphically.

52. $h(x) = \frac{4}{x^2}$

Exer. 59-68: Find the inverse function of f algebraically. Use a graphing utility to graph both f and f^{-1} in the same viewing window. Describe the relationship between the graphs.

65. $f(x) = \sqrt{4 - x^2}$ $\quad 0 \leq x \leq 2$

Exer. 81-88: Use the graphs of $y = f(x)$ and $y = g(x)$ to evaluate the function.

82. $g^{-1}(0)$