006

Thursday, September 11, 2014

Combinations of Functions

Given: $f(x) = 3x^2 - 10$ and $g(x) = \sqrt{x+4}$
Domain of $f = (-\infty, \infty)$ Domain of $g = [-4, \infty)$

Sum

1. $(f + g)(x) = f(x) + g(x) = 3x^2 - 10 + \sqrt{x+4}$

2. $(f + g)(5) = 3(25) - 10 + \sqrt{9} = 75 - 10 + 3 = 68$

3. Domain of $(f + g)$ is the Intersection of the Domains of f and g: $[-4, \infty)$

Difference

4. $(g - f)(x) = g(x) - f(x) = \sqrt{x+4} - 3x^2 + 10$

5. $(g - f)(3) = \sqrt{7} - 27 + 10 = -17 + \sqrt{7}$

6. Domain of $(g - f)$ is $[-4, \infty)$

Product

7. $(fg)(x) = f(x)\, g(x) = (3x^2 - 10)\sqrt{x+4}$

8. $(fg)(12) = f(x)\, g(x) = (3(144) - 10)\sqrt{16} = (432 - 10)(4) = 422 \bullet 4 = 422*4 = 1688$

Quotient

9. $\left(\frac{g}{f}\right)x = \frac{g(x)}{f(x)} = \frac{\sqrt{x+4}}{3x^2 - 10}$, $f(x) \neq 0$

10. $\left(\frac{g}{f}\right)(0) = \frac{\sqrt{4}}{-10} = -\frac{1}{5}$

Composition

11. $(f \circ g)(5) = f(g(5)) = f(3) = 27 - 10 = 17$

12. $(g \circ f)(1) = g(f(1)) = g(-7)$ which is undefined

13. Domain of $(f \circ g)$ is all x in the domain of g such that
    g(x) is in the domain of f

14. Given: $f(x) = \sqrt{-x^2 + 12x - 20}$ and $g(x) = \sqrt{16 - x^2}$
    Find the Domain of $(f \circ g)(x)$

    Domain of g(x) = [-4, 4]
    Range of g(x) = [0, 4]

    Use [0, 4] from the Domain [2, 10] of f(x)

    Domain of $(f \circ g)(x)$ = only elements from [2, 10] that are in [0, 4]

The outcomes of g (its Range) using inputs in [0, 4] must be in [4, 0]

15. Given: $f(x) = x^2 - 4$ and $g(x) = \sqrt{9 - x^2}$ Find the Domain of $(f \circ g)(x)$
    $f(g(x)) = f\left(\sqrt{9 - x^2}\right) = 9 - x^2 - 4 = -x^2 + 5$
    The Domain of g is [-3, 3], so the range of g is [0, 3]
    f can only use values on [0, 3] as its Domain
    The Range of f(g(x)) is [-4, 5]

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01.05 Combinations of Functions
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Exer. 5-12: Find (a) (f + g)(x), (b) (f - g)(x), (c) (fg)(x) (d) (f/g)(x). What is the domain of f/g?

6. $f(x) = 2x - 5,\ g(x) = 1 - x$

(a) $(f + g)(x) =$

(b) $(f - g)(x) =$

(c) $(fg)(x) =$

(d) $(f/g)(x) =$

Exer. 13-26: Evaluate the indicated function for $f(x) = x^2 - 1$ and $g(x) = x - 2$ algebraically. If possible use a graphing utility to verify your answer.

13. $(f + g)(3) =$

16. $(f + g)(1) =$

18. $(fg)(-6) =$

25. $\left(\frac{f}{g}\right)(-t) =$

Exer. 35-38: Find (a) $f \circ g$, (b) $g \circ f$, and, if possible, (c) $(f \circ g)(0)$.

35. $f(x) = x^2,\ g(x) = x - 1$ (a) (b) (c)

37. $f(x) = 3x + 5,\ g(x) = 5 - x$ (a) (b) (c)

Exer. 39-48: Determine the domain of (a) f, (b) g, and (c) $f \circ g$. Use a graphing utility to verify your results.

39. $f(x) = \sqrt{x+4},\ g(x) = x^2$ (a) (b) (c)

40. $f(x) = \sqrt{x+3},\ g(x) = \frac{x}{2}$ (a) (b) (c)

42. $f(x) = x^{1/4},\ g(x) = x^4$ (a) (b) (c)

43. $f(x) = \frac{1}{x},\ g(x) = x + 3$ (a) (b) (c)

45. $f(x) = |x - 4|,\ g(x) = 3 - x$ (a) (b) (c)

Exer. 65-72: Find two functions f and g such that $(f \circ g)(x) = h(x)$. (There are many correct answers.)

66. $h(x) = (1 - x)^3$

68. $h(x) = \sqrt{9 - x}$