Determine whether each is a Calculus or Pre-Calculus Exercise.

1. The number of gallons, g, in a tank at time, t, is given by $g(t) = 2t^2 + 3t$. Find the amount of water added to the tank from time t = 2 to time t = 5.

   51 Gallons – This is a pre-calculus problem

2. An object is moving in a straight line so that the distance, s, in miles from 0 at time, t, in hours is given by the function, $s(t) = 2t^2 + \sin t$. At what value of time t, is the distance, s, at 20 miles?

   3.164 hours – This is a pre-calculus problem

3. From #2, find the time at which the vehicle is traveling at 20 mph.

   4.943 hours – This is a calculus problem

4. Find the distance traveled in 15 seconds by an object moving with a velocity of $v(t) = 20 + 7\cos t$ feet per second.

   A calculus problem with insufficient information. We need to know which 15 seconds.

Before moving on, I want to invite you to become familiar with the following website:

http://www.wolframalpha.com

Definition of Limit: Let f be a function defined on an open interval containing c (except possibly at c) and let L be a real number. The statement $\lim_{x \to c} f(x) = L$ means that for each $\varepsilon > 0$, $\exists$ a δ such that if $0 < |x - c| < \delta$, then $|f(x) - L| < \varepsilon$.

$g(x) = 4x - 7$. Find $\lim_{x \to 2} g(x)$ and prove that your limit is correct.

$\lim_{x \to 2} 4x - 7 = \boxed{1}$

Proof:
Because of the definition of limit, we must Show that:
$\forall\, \varepsilon > 0, \exists\, \delta > 0 \ \ni.\ 0 < |x - 2| < \delta \rightarrow |4x - 7 - 1| < \varepsilon$.

$|4x - 7 - 1| < \varepsilon$

$\rightarrow |4x - 8| < \varepsilon$

$\rightarrow 4|x - 2| < \varepsilon$

$\rightarrow |x - 2| < \frac{\varepsilon}{4}$

Choose $\delta = \frac{\varepsilon}{4}$

We must now show that $|x - 2| < \delta \rightarrow |4x - 8| < \varepsilon$.

$|x - 2| < \delta$

$\rightarrow |x - 2| < \frac{\varepsilon}{4}$

$\rightarrow 4|x - 2| < \varepsilon$

$\rightarrow |4x - 8| < \varepsilon$

Find the limit L by Direct Substitution. Then find $\delta > 0 \ \ni.\ |f(x) - L| < 0.01$ whenever $0 < |x - c| < \delta$.

$\lim_{x \to 2} (x + 2)$

$\lim_{x \to 2} (4 - \frac{x}{2})$

$\lim_{x \to 5} (x^2 + 4)$

Use the ε-δ definition to prove that the limit is L.

$\lim_{x \to 2} (x + 2)$

$\lim_{x \to 2} (\sqrt{x})$

$\lim_{x \to 2} (x^2 + 1)$

Sometimes a table of values can give a hint for the value of a limit

What does your calculator suggest as the limits of the next 2 expressions? The calculator in this case is NOT a PROOF.

$\lim_{x \to 0} \frac{\sin x}{x}$

x	-0.1	-0.01	-0.001	0.001	0.1
f(x)					

$\lim_{x \to 0} \frac{\cos x - 1}{x}$

x	-0.1	-0.01	-0.001	0.001	0.1
f(x)					

The above are Special Limits which we will prove at a later time, but remember them now.

Special Limits

a. $\lim_{x \to 0} \frac{\sin x}{x} = 1$

b. $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

So far, we have evaluated limits by Direct Substitution, that is, simply plugging values into the original expression.

Consider $\lim_{x \to 2} \frac{x^2 + x - 6}{2x^2 - 5x + 2}$. Direct substitution will not work.

Factoring can lead to an answer: $\lim_{x \to 2} \frac{x^2 + x - 6}{2x^2 - 5x + 2} = \lim_{x \to 2} \frac{(x+3)(x-2)}{(2x-1)(x-2)} = \lim_{x \to 2} \frac{x+3}{2x-1} = \frac{5}{3}$

Consider: $\lim_{x \to 5} \frac{\sqrt{x-4} - 1}{x - 5}$ Again, Direct Substitution will not work.

In this case, rationalizing the numerator can help.
Multiply top and bottom by the conjugate of the top.

$$\lim_{x \to 5} \frac{\sqrt{x-4} - 1}{x - 5} = \lim_{x \to 5} \frac{\sqrt{x-4} - 1}{x - 5} \bullet \frac{\sqrt{x-4} + 1}{\sqrt{x-4} + 1} = \lim_{x \to 5} \frac{x - 5}{(x-5)(\sqrt{x-4} + 1)} = \lim_{x \to 0} \frac{1}{\sqrt{x-4} + 1} = \frac{1}{2}$$

So now, we have 3 ways of evaluating limits analytically:

1 Direct Substitution
2 Factoring
3 Rationalyzing the Numerator

AP Calculus BC 1 Assignment 002

01.03 Evaluating Limits Analytically

Page 67, #'s 23c, 26c, 37a, 40a, 44a, 46, 55, 63, 67, 70, 71, 77, 83, 84